Complex Algebraic Geometry
MSRI Publications
Volume **28**, 1995

Preface

The 1992/93 academic year at the Mathematical Sciences Research Institute was devoted to Complex Algebraic Geometry. The cochairs of the organizing committee were Herb Clemens and myself. Early on we decided that the activities would be centered around themes, each month having its own theme. In organizing each theme, we relied on the help of experts in that area. The success of the year depended very much on their energy and enthusiasm. The following is the list of topics and of the organizers:

September: *Algebraic cycles* (A. Beilinson, W. Fulton)
October: *Vector bundles* (R. Lazarsfeld)
November: *Higher-dimensional geometry* (J. Kollár, S. Mori)
December: *Curves, abelian varieties and their moduli* (A. Beauville, J. Harris)
January: *Surface theory, classical projective geometry* (R. Friedman, J. Harris)
February: *Topology of moduli spaces* (E. Arbarello)
March: *Enumerative and computational algebraic geometry* (W. Fulton)
April: *Crystalline methods and Hodge theory* (A. Beilinson, A. Ogus)
May: *Singularity theory and Hodge theory* (M. Green, J. Steenbrink)

There were also four short workshops, which attracted many participants and helped considerably in communicating the new directions in algebraic geometry to a large audience. The following is the list of workshops held at MSRI:

Algebraic cycles (A. Beilinson, W. Fulton)
Higher dimensional geometry (J. Kollár, S. Mori)
Curves, abelian varieties and their moduli (A. Beauville, J. Harris)
Enumerative geometry and physics (W. Fulton)

In conjunction with the Complex Algebraic Geometry Year at MSRI there were three additional workshops organized at nearby Universities:

Vector bundles, UC Los Angeles (R. Lazarsfeld)
Crystalline methods and Hodge theory, UC Berkeley (A. Ogus)
Hodge theory and singularities, UC Riverside (M. Green, Z. Ran, J. Steenbrink)

We would like to thank the organizers of the workshops for their valuable contributions to the scientific success of the special year.

The aim of this volume is to collect survey articles that give a good idea of the direction of the special year and of the workshops. We were very fortunate to have had the special year at a time when algebraic geometry was undergoing a major change. To put it succinctly, algebraic geometry has opened up to ideas and connections from other fields that have traditionally been far away. The articles of this volume represent this change of direction very well.

(As the reader may have noticed, the initials of all the contributors of this volume fall in the first half of the alphabet. What is the explanation of this overrepresentation? Is this one of the new directions? What, if any, remedies should be applied?)

Arapura surveys recent results connected with the fundamental groups of smooth projective varieties. Traditionally the fundamental group was viewed as a strange invariant of a variety, not related to the usual concepts of algebraic geometry, except in the case of curves. Recently there has been a complete turnaround, and now we understand that the fundamental group relates to diverse questions in subtle ways, many of them still poorly understood. This direction generated a lot of interest at the time of the special year and since. One of the most exciting developments is the connection with the theory of harmonic maps.

Vector bundles on curves are very familiar objects. A few years ago interest in them was rekindled by the Verlinde conjectures coming from theoretical physics. These conjectures give an explicit formula for the Riemann–Roch theorem for the moduli of vector bundles. The intrinsic beauty and accessibility of these formulas suddenly generated a lot of activity. At times it seemed that everyone was trying to understand their behaviour. This high level of interest yielded several solutions from different points of view. The varied approaches turned out to complement each other very nicely. This led to further development in the theory of moduli spaces, where methods from algebraic geometry and analysis of geometric PDE's can be used side by side. This is the subject of the survey article by Beauville.

One of the major internal developments of algebraic geometry in the last decade has been the emergence of higher-dimensional birational geometry, and especially Mori's minimal model program. After a very intense decade of perfecting various techniques, the direction of the field turned toward applying the ideas and methods of the program to other questions. The article by Corti reviews the main steps and techniques of the program, and then goes on to explain recent applications to the study of the moduli of surfaces, to the Nöther–Fano–Iskovskikh theory of birational maps, and to the effective base point freeness question.

The latter topic is taken up in more detail in the survey by Ein. This is an especially well chosen topic, since here one can illustrate the general methods of the minimal model program in a very simple and clear situation. These applications are rather surprising and they opened up many new directions. Recent extensions of the theory to vector bundles hold a lot of promise.

Abelian varieties are one of the oldest and most enduring topics of algebraic geometry, and this field always comes up with new perspectives. The short survey by Debarre guides the reader to many of them. One of the most surprising new developments is the realization that the lattices of Jacobians have very strong metric properties. It still remains to be seen how these can be understood in terms of algebraic geometry.

Spectral covers originated in the study of vector bundles of curves. They have found connections with the study of vector bundles on higher-dimensional varieties, with the theory of fundamental groups of varieties and with integrable Hamiltonian systems. The survey by Donagi is devoted to their study.

Another classical topic that always finds a new life is the theory of the moduli of curves. The study of this moduli space can be approached from many angles. The connection with the Torelli group is the guiding principle of the article by Hain. This method reveals profound connections with the topology of Riemann surfaces, with the cohomology theory of discrete groups and with Hodge theory.

The last contribution of the volume by Mukai provides a connection with many topics. The theory of special vector bundles on curves can be viewed as a natural development starting with questions about the moduli of curves. It also relates to the questions of Verlinde type. Most surprisingly, several of the resulting moduli spaces give interesting examples of Fano varieties. This led to a deeper understanding of the classification of Fano threefolds.

We hope that, by collecting these lectures in a single volume, we can convey at least some of the sense of the new directions that started to develop during the special year at MSRI.

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