Games of No Chance
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Games with Infinitely Many Moves and Slightly Imperfect Information

(Extended Abstract)

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D. A. Martin in 1975 showed that all Borel Games with perfect information are determined. Question: are all Borel games with slightly imperfect information determined?

Let A, B be finite nonempty sets, let $C = A \times B$, and let W be the set of all infinite sequences $w = \{c_1, c_2, \dots\}$ from C. Any subset S of W defines a game $G(S)$, whose n-th move, for $n = 1, 2, \dots$, is played as follows: Player I chooses $a_n \in A$ and, simultaneously, Player II chooses $b_n \in B$. Each player is then told the other's choice, so that they both know $c_n = (a_n, b_n)$.

Player I wins $G(S)$ just if the play $w = \{c_1, c_2, \dots\}$ is in S. We say that $G(S)$ is *determined* if there is a number v such that, for every $\varepsilon > 0$,

(a) Player I has a (random) strategy that wins for him with probability at least $v - \varepsilon$ against every strategy of Player II, and

(b) Player II has a (random) strategy that restricts his probability of loss to at most $v + \varepsilon$ against every strategy of Player I.

If S is *finitary*, i.e., depends on only finitely many coordinates of w, then $G(S)$ is a finite game, and the von Neumann minimax theorem says that $G(S)$ is determined.

If S is *open*, i.e., the union of countably many finitary sets, then it is well-known, and not hard to see, that $G(S)$ is determined (and that Player II has a good strategy).

If S is a G_δ*-set*, that is, the intersection of countably many open sets, again $G(S)$ is determined, but the calculation involves countable ordinal calculations. This complexity is probably necessary as, with a natural coordinatization of G_δ-sets, the value $v(S)$ is not a Borel function of S. Whether all $G_{\delta\sigma}$ games $G(S)$ are determined is not known.

Our restriction to finite A and B is essential. For A and B countable, the game “choosing the larger integer” is a special case that is *not* determined.

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