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Championship-Level Play of Domineering

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Abstract. A single-elimination Domineering tournament was held at the MSRI meeting, with a \$500 purse. This is an analysis of the finals of that tournament, in which Dan Calistrate defeated David Wolfe by three games to one. An algebraic notation for commenting games is introduced.

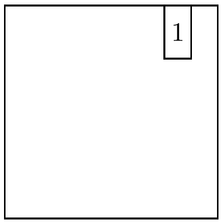
Domineering is a game played on subsets of the square lattice by two players, who alternately remove connected two-square regions (dominoes) from play. Left may only place dominoes vertically; Right must play horizontally. The normal win-condition applies, so that the first player unable to move loses.

It is difficult to analyse general Domineering positions, even for quite small boards. One way to gain insight into the nature of the problem is to watch actual games between expert players. To determine who were the strongest players available, an open-registration tournament was held. To insure that the players gave proper consideration to their play, a prize of \$500 was awarded the winner.

The finalists were Dan Calistrate of Calgary and David Wolfe of Berkeley. The format for the final was to play two games, each player taking the first turn once; if the series was split, two more games would be played, and so on until one player won both games of a round. As in chess, it would be expected that one set of pieces would provide an advantage, therefore winning with the favoured set would be like holding serve in tennis. In the event, the first round produced two first-player wins, and Calistrate won both second-round games.

An 8×8 board was selected as large enough to be beyond the range of current analysis, and therefore apt to provide an interesting game. It seems quite likely that analysis of the general position is genuinely hard, so that the solution of the 8×8 board would simply necessitate a move to 9×9 or 10×10 to retain interest.

Because of symmetry, the square board must be either a first-player or a second-player win (as opposed to a win for Left or a win for Right). Indeed, because of this symmetry, we can always orient the board so that the player who starts is considered to be Left; we have adopted this convention for clarity. The 5×5 board is known to be a second-player win (that is, it has value 0; see *On Numbers and Games* [Conway 1976]). Conway observes that for larger boards "0 is an infinitely unlikely value", because this value would only obtain if Left's *best* move is to a position of net value to Right. This becomes less and less likely the more options Left is given.

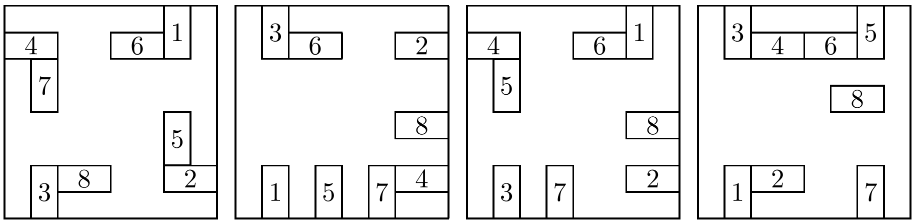


Furthermore, the actual opening move generally agreed on as strongest appears to be quite strong. It leaves the board in the position shown at the right, which is at least as good for Left as $1-$ (the 8×8 board with a 2×2 bite out of the corner). That board is also likely to be of net value to the first player, but presumably has value less than 1.

A word about notation: we have numbered the dominoes in the order in which they were placed on the board. Thus Left (vertical) dominoes have odd numbers. When it is possible to compute the game-value of a contiguous region, we will write it in.

In actual play, the game appears to go through three stages: an early game, a middle game, and an end game. Nearly all of the interest appears to be in the middle game. The early game lasts about eight moves (half-turns), and consists of corner moves similar to the opening explained above. It appears that experts cannot distinguish between the relative values of these openings, seeing them rather as variants. The choice of which variant to play may be as much psychological as anything, trying to guess a terrain that may be less familiar for the opponent. Unfortunately, we do not have as rich a vocabulary for our textbook openings as does the chess world, but the actual positions reached in the championship games were as shown in Figure 1.

In the early game, every move reserves a future move for the same player by creating a space in which only that player can place a domino. This suggests that Domineering on a spacious board is very hot, explaining the difficulty of a thorough analysis.



Game 1 Game 2 Game 3 Game 4

Figure 1. Position at end of opening for each of the four games in the finals. In games 1 and 3, Wolfe is Left (vertical) and Calistrate is Right (horizontal). In games 2 and 4, the roles are reversed.

In the middle game, in the space of about six moves, the board is converted from the relatively large open space of the "textbook" opening into a fractured board consisting entirely of numbers and switches. Since such a position is easy to analyse and to play flawlessly, the end game need not be played out. As the large, difficult to analyse region of the board is sectioned, it is rarely broken into two hot regions; therefore it is usually apparent in which component a player should play. Finding the correct move can be subtle, and a mistake is often fatal, since the game is really decided in about six moves.

Since the game is presumably a win for Left, and since it is decided in the space of six moves, it is very unlikely that control will be exchanged even twice. Therefore, Left wins exactly those games in which Left makes no errors. Right wins games in which Left makes an error, and Right successfully exploits it. These games can be viewed as "service breaks". Since any winning move is a good move, wins for Left consist entirely of good moves. Wins for Right contain exactly one bad move. So there is only one bad move to be pointed out in these four games.

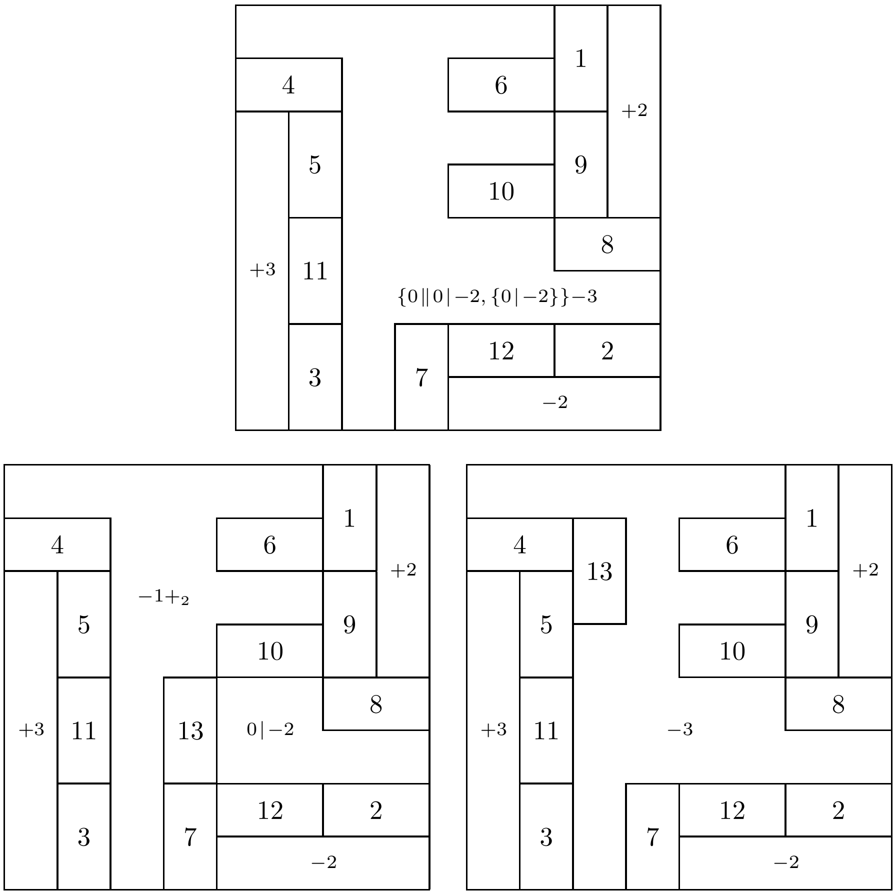
In one sense, the onus is on the first player to initiate the transition to the middle-game. Imagine the board divided into 2×2 squares. If no dominoes ever cross the borders between these squares, the second player will be able to win by claiming half of the squares. The early game consists of moves that respect these borders, so the transition to the middle game is the breaking of this respect, and it is the first player who benefits. Note that the opening of game 4 is an odditiy in this respect, in that it is the second player who breaks symmetry (at move 8).

Game 3, the only one that led to a second-player win, breaks this expectation in the other sense: the first player *failed* to break symmetry. Since this was the "service break" game, it is the only one in which we can definitively say that a mistake was made, and it thus proves interesting to analyse. After twelve moves, the situation shown at the top in Figure 2 was reached.

In actual play, Left played as in the bottom left diagram of Figure 2. It looks like a reasonable move, since as it continues the paradigm of reserving a free move for Left; but it leaves a devastating future move for Right in the six-square region. The correct move, the only winner, is not easy to spot; it is shown on in the bottom right diagram of Figure 2.

Notice that this fulfils our expectation that this is the moment at which the first player must break out of the division into 2×2 cells.

The other three games simply demonstrate the considerable latitude that the first player has for securing the (assumed) win. Figure 3 shows the positions reached just as they become susceptible to analysis. Figure 4 shows the games again as they cool down to sums of numbers and switches. In game 2, Left's move 13 is indeed the optimal move (to $7 \,|\, \frac{11}{2} \,\|\, 5, \{5, 4\}$). Right's reply selects the option $+5$.



Actual move Correct move

Figure 2. Left's fatal mistake in Game 3: Wolfe is Left (vertical) and Calistrate is Right (horizontal).

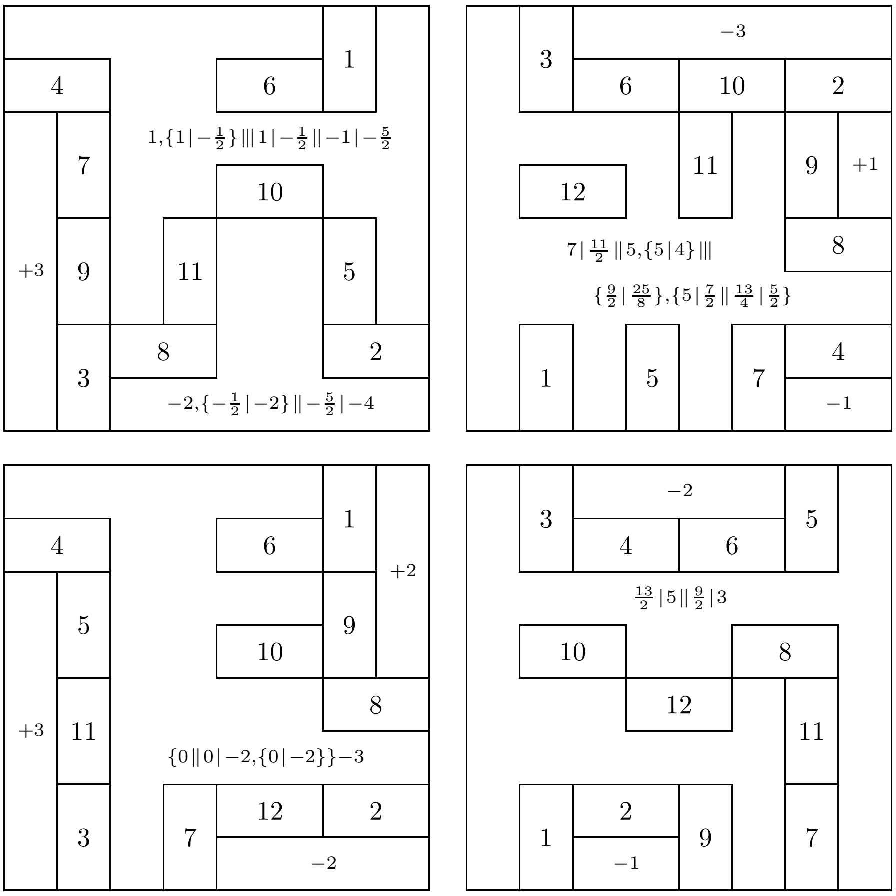


Figure 3. Positions reached in games 1–4 just as they become susceptible to analysis.

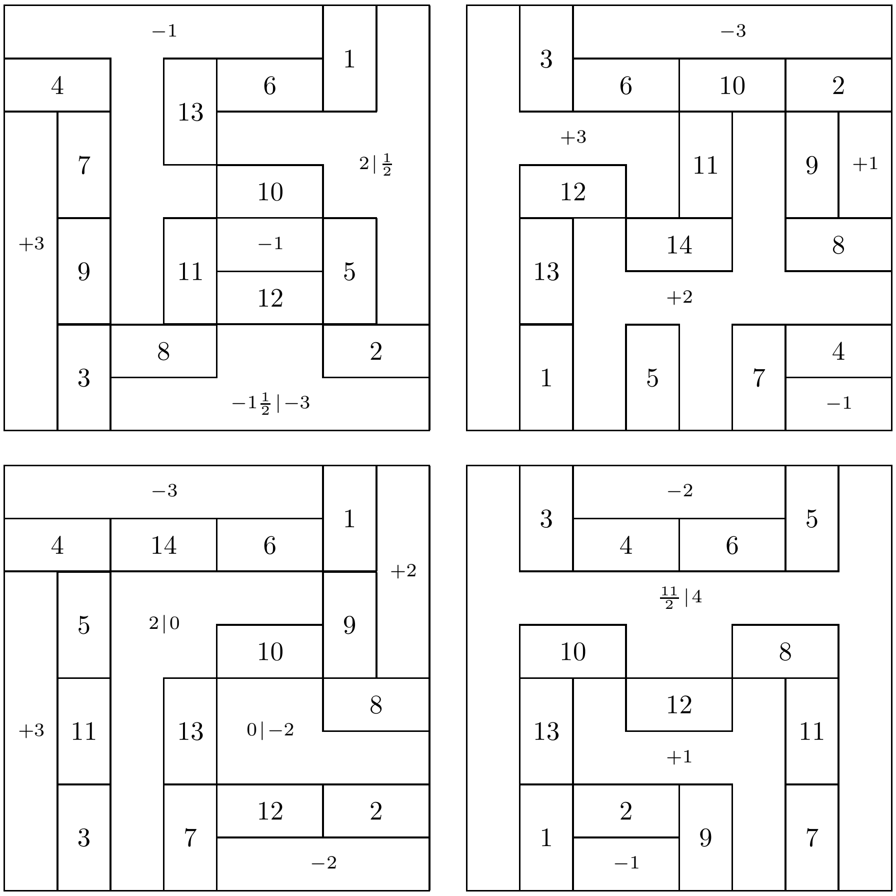


Figure 4. Positions reached in games 1–4 just as they cool down to sums of numbers and switches.

Values were computed using David Wolfe's *games* software. Boards were typeset using Jennifer Turney's awk scripts for domineering, which produce LaTeX code.

Reference

[Conway 1976] J. H. Conway, *On Numbers And Games*, Academic Press, London, 1976.

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