Comparison Geometry
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Preface

One of the main activities during the 1993–94 special year in differential geometry at MSRI was focused on the subject baptized "Comparison Geometry" during the planning phase of the workshops.

Although a name has been lacking for this beautiful and most geometric branch of riemannian geometry, its history can be traced back to the nineteenth century. It did not take root, however, until the 1930's, through the work of H. Hopf, Morse–Schoenberg, Myers, and Synge. The real breakthrough came in the 1950's with the pioneering work of Rauch and the foundational work of Alexandrov, Toponogov and Bishop. Since then, the simple idea of comparing the geometry of an arbitrary riemannian manifold with the geometries of constant curvature spaces has seen a tremendous evolution: first in conjunction with Morse theory and convexity, then with critical point theory for distance functions, and most recently with the Gromov–Hausdorff topology on spaces of riemannian manifolds, and the geometry of singular spaces. As a result, our understanding of relations between the geometry and topology of riemannian manifolds has gained tremendous breadth and consists no longer of just a short string of pearls.

At the outset it is worth mentioning that the flavor and character of problems and techniques related to upper rather than lower curvature bounds to a large extent are remarkably different. This volume is an up-to-date reflection of the above mentioned development regarding spaces with lower, or two-sided, curvature bounds. The subject of manifolds with negative or nonpositive curvature, with its ramifications to dynamics and number theory, is not represented here.

The content of the volume reflects some of the most exciting activities on Comparison Geometry during the year, and especially of the workshop devoted to the subject. As a consequence, the book features survey articles (by Abresch and Meyer, Anderson, Colding, Greene, Otsu, Petersen, and Zhu) and research articles (by Perelman and Petrunin). Each of the survey articles stems from recent interesting developments concerning either classical or more recent important problems and two of them are lecture notes from one-quarter courses taught by the authors. Complete proofs are often provided, and in one case a new unified strategy is presented and new proofs are offered. which article?

We hope that this volume will be a valuable source for those who wish to learn about this beautiful subject and contribute to it.

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