Flavors of Geometry
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Preface

> . . . βούλεσθ', εἶπεν, ἐπεὶ λόγοι περὶ θεῶν γεγόνασιν,
> ἐν τοῖς Πλάτωνος γενεθλίοις αὐτὸν Πλάτωνα κοινωνὸν
> παραλάβωμεν, ἐπισκεψάμενοι τίνα λαβὼν γνώμην
> ἀπεφήνατ' ἀεὶ γεωμετρεῖν τὸν θεὸν; εἴ γε δὴ θετέον εἶναι
> τὴν ἀπόφασιν ταύτην Πλάτωνος.
>
> 'Εμοῦ δὲ ταῦτ' εἰπόντος ὡς γέγραπται μὲν ἐν οὐδενὶ
> σαφῶς τῶν ἐκείνου βυβλίων, ἔχει δὲ πίστιν ἱκανὴν καὶ
> τοῦ πλατωνικοῦ χαρακτῆρός ἐστιν . . .
>
> — Plutarch, *Quaest. Conv.* VIII.2, in his *Moralia*

God is always doing geometry. What Plato meant by this is no clearer now than it was in the first century A.D., when the dinner conversation quoted above (which I'm about to paraphrase) took place. Diogenianus, one of the guests, recalls that it is Plato's birthday and proposes to "bring him into the conversation" by debating the meaning of this nugget—"if indeed the statement is Plato's." Plutarch, who knew a lot about such things, replies that although the quote does not appear explicitly in any of Plato's books, it is well enough attested, and it is in character.

There follow the opinions of the three remaining guests, and finally that of Plutarch. Drastically abridged, they are:

- Geometry focuses the mind on the abstract rather than on the sensorial.
- Geometry is divine in that "geometric proportion . . . befits a moderate oligarchy or a lawful monarchy . . . it distributes to each according to his worth," whereas arithmetic proportion is egalitarian.
- The essence of geometry is boundaries, so God does geometry when he bounds matter to create.
- Geometry intervenes when proportion and measure and number are used to order chaotic nature.

Although encompassing, these are not entirely persuasive explanations; make of them what you will. I bring them up largely for fun, but also to illustrate the breadth of interpretation that the notion of geometry has traditionally enjoyed, and so to defend my choice of title for this volume. One might consider also Klein's definition in his Erlangen program. And, while the study of complex dynamics in one or more variables involves tools from analysis, topology, and so on, a large part of the motivation is geometric: we want to understand the qualitative and spatial behavior of orbits.

* * *

This book is based on introductory lecture series that took place at MSRI in 1995 and 1996, aimed at graduate students and mathematicians in all fields.

The first series of lectures ran from January 9 to 20, 1995, as a prelude to the Spring 1995 program in Complex Dynamics and Hyperbolic Geometry. It consisted of three courses of five lectures each:

- *Hyperbolic Geometry* by James Cannon;
- *Conformal Dynamics on the Riemann Sphere* by John Hubbard;
- *Complex Dynamics in Several Variables* by John Smillie.

Of these, the first and last are included in this volume; notes for Smillie's lectures (pp. 117–150) were written by Gregery Buzzard, and those for Cannon's lectures (pp. 59–115) primarily by him, with the help of Bill Floyd, Richard Kenyon, and Walter Parry.

The second series of lectures ran from January 29 to February 9, 1996, in conjunction with the Spring 1996 program on Convex Geometry and Geometric Functional Analysis. Four courses of 3–4 lectures each were offered:

- *Classical Convex Geometry* by Keith Ball;
- *Concentration of Measure in Geometry* by Gideon Schechtman;
- *Spherical Sections of Octahedra* by Joram Lindenstrauss;
- *Random Methods for Volume Computation* by Béla Bollobás.

Keith Ball kindly agreed to merge the first three courses into one set of notes (pp. 1–58), so that Schechtman's and Lindenstrauss's material is now, after considerable transmutation, integrated with the more basic material of Ball's lectures. Bollobás wrote up his own lectures (pp. 151–182).

I've arranged the four contributions from easier to harder, but relative difficulty depends somewhat on one's background, so you should browse. Each set of notes starts with elementary concepts, proceeds through highlights of the field, and concludes with a taste of advanced material. Some math undergraduates will find most of Ball's and Cannon's contributions, and at least the beginning of the others, perfectly accessible.

For ease of reference I have supplied an index for each set of notes. Any shortcomings the indexes may have are to be blamed on me, not on the authors.

Silvio Levy
Berkeley, February 1997

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Note on MSRI Programs

The introductory courses and workshops that gave rise to this book were part of the plan, advanced by the MSRI directorate during the last several years, of intensifying the Institute's effectiveness and outreach by means other than research in core mathematics (which nonetheless remains the center of our activities). Some others among these outreach efforts:

- Conversations between Researchers and Teachers, a series of presentations and discussions bringing together two groups that rarely mix: high-school math teachers and research mathematicians.
- A conference on math visualization, one on The Future of Mathematical Communication, and one on The Future of Mathematical Education at Research Universities.
- The Fermat Fest, which explained to a lay public of 1000 the meaning and basic ideas of Fermat's Last Theorem.
- Numbers in Action, another event for the general public focusing on number theory.
- A program for broadcasting talks on the MBone (the Internet multicast backbone) and for helping other sites obtain access to the MBone.
- The Conference for African American Researchers and the Julia Robinson Celebration of Women in Mathematics Conference.
- Research workshops and programs on nontraditional topics, such as financial mathematics and combinatorial game theory.

Details about these events and programs, as well as most other MSRI activities, can be found at http://www.msri.org.