Tight and Taut Submanifolds
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# Remembering Nicolaas Kuiper

THOMAS F. BANCHOFF

Tight and taut immersions are a living and growing part of contemporary mathematics largely due to the legacy of Nicolaas Kuiper. He made central contributions to many different areas of mathematics during his long and productive career, but it is in tight and taut immersions that his geometric style showed forth in a special way. In that subject, his personal enthusiasm and extraordinary geometric insight combined to bring forth examples and theorems of great conceptual and visual appeal. He delighted in discovering new phenomena, and in presenting his examples using sketches and in cardboard or wire-frame models. He found surprising connections among apparently unrelated areas of mathematics, creating entirely new methods for handling a range of geometric structures: analytic, differentiable, once-differentiable, combinatorial, and topological. He was the first to appreciate the essentially geometric character of tightness, exploiting the relationship between the minimal total curvature condition for smooth submanifolds and critical point theory so that the notion could be extended to non-smooth objects. He guided generations of mathematicians who have followed his lead.

In several other subjects his contributions were necessarily abstract, for example in the embedding theorems he produced with John Nash, or the surprising result that the unit sphere in Hilbert space is contractible. Often he would listen to lectures on a new subject, read about it and study it, and come up with a crucial insight that no one else was close to realizing. He would then leave the field to other mathematicians, encouraging their efforts. He was particularly supportive of young colleagues from many different countries, especially while he was director of the Institut des Hautes Études Scientifiques.

What was special about the theory of tight and taut immersions that kept bringing him back to the subject over a period of more than thirty years? Certainly it had to do with the original examples that kept appearing, illuminating new parts of the subject. Many of these phenomena he discovered himself, but equally important were the examples found by others, which he then helped to bring into full flower. He had such a varied background that he could often see some potential relationships that just about anyone else would have missed.

Who else would have recognized the central significance of the Veronese surface as the unique smooth tight surface in five-dimensional space? The proof of that result was based on theorems in projective differential geometry that almost no one knew about, but he did, at just the right time.

Some mathematicians are renowned because they are the first to arrive at a goal that many others are seeking at the same time. Others are remarkable because they find things no one else even thought of looking for. Nico Kuiper will be remembered as one of the most original mathematicians of his time.

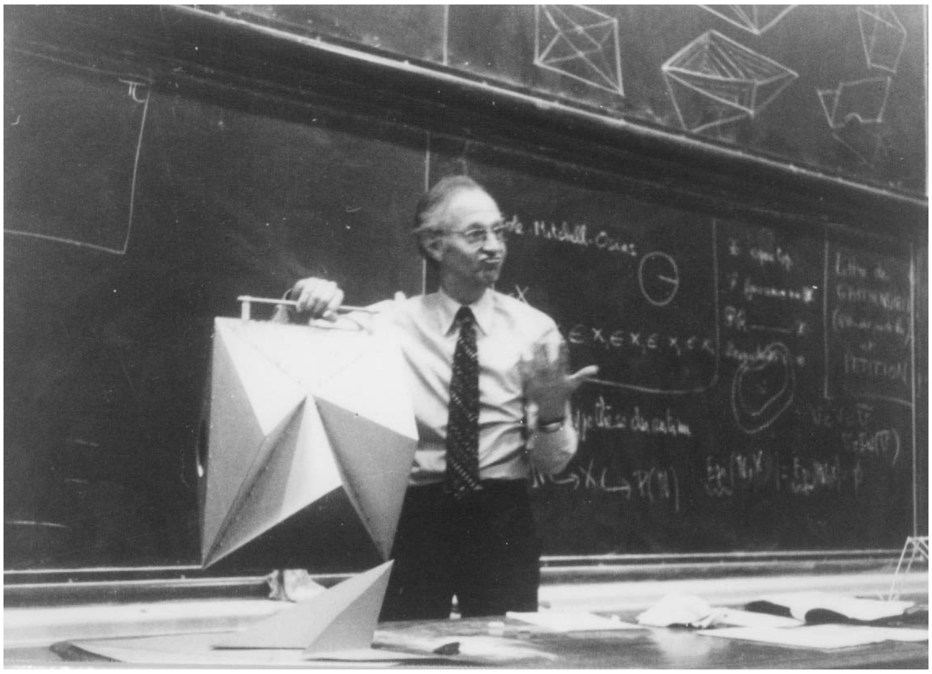
In 1946, Kuiper received his Ph.D. at the University of Leiden working under the direction of Willem van der Woude in the field of classical differential geometry. He then had the chance to come to the United States, first at the University of Michigan where he met Raoul Bott and his student Steve Smale, and then at the Institute for Advanced Study for a crucial interaction with Shiing-Shen Chern. It is easy to see that even at this early stage in his career, Kuiper had a characteristic way of working—he would pay attention to a result or an approach of another mathematician and find that he was rephrasing the concepts in his mind, asking new questions, and more often than not coming up with a fresh insight. It is not surprising that he became a coauthor of a great many papers over the years.

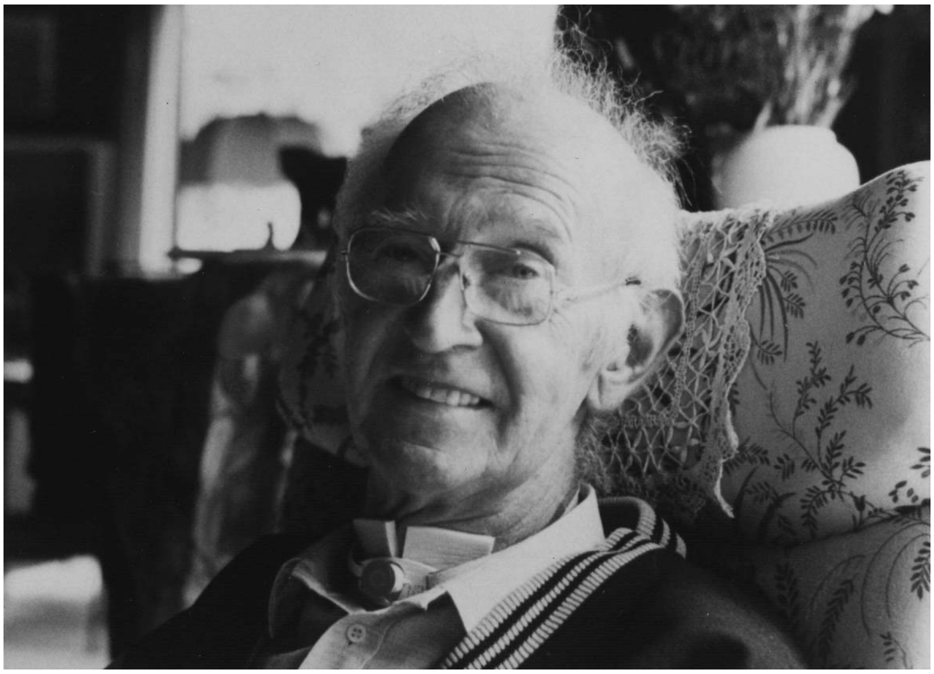
During the 1950's he taught mathematics and statistics at the Landbouwhogeschool (Agricultural Institute) in Wageningen, contributing a number of geometric insights to the theory of design of experiments. In the 1960's, he was professor of pure mathematics at the University of Amsterdam, concentrating primarily in the fields of differential geometry, differential topology, and algebraic topology, and nurturing a number of doctoral students and post-doctoral visitors. From 1971 to 1985, he was Director of the Institute des Hautes Études Scientifiques at Bures-sur-Yvette near Paris, where he exercised leadership in the world community of mathematicians. He was made a Knight of the Order of the Golden Lion by the Dutch Government and he was a Chevalier in the French Legion of Honor. In 1984, he received an honorary degree from Brown University. After his retirement, he remained in France until 1991, when he returned to live in the Netherlands. He continued to participate in mathematical colloquia at the University of Utrecht.

Nico Kuiper was only 74 years old when he died on December 12, 1994 after a year-long illness that had taken away his strength but not his love for mathematics. On a personal note, I was privileged to be able to visit him and his wife Agnete at their home in the Netherlands after the International Congress of Mathematicians in Zürich in August of that year, and I could see how difficult it was for him to be confined by his illness. He had attended so many Congresses, and had taken a leadership role at an officer of the International Mathematical Union. Now he could only hear reports of the new developments in the many subjects in which he had made key contributions.

One result that pleased him especially was a breakthrough in the theory of tight immersions. Thirty years ago, he had singled out as a special challenge the question of finding a tight immersion of the real projective plane with one handle, the sole remaining case for surfaces in three-space. He and I had traded letters on this subject for years, and more than once one of us would present elaborate drawings of a purported solution, only to follow it the next day by a "disregard previous letter" after finding an unallowable local self-intersection. Just two years earlier, François Haab had shown that there was no tight smooth immersion of this surface into three-space, and Nico himself had been instrumental in working with Haab to extend these results. He was surprised and delighted to learn about the discovery by Davide Cervone of a tight polyhedral immersion of this surface into three-space. It is too bad that he did not have access to the Internet so he could work interactively with this beautiful example, but he did draw the diagrams based on the coordinate description. In previous years, he had only been able to look for examples with small numbers of vertices, and it impressed us both to see how computer graphics had increased the opportunities to find and manipulate complicated new examples. (This example and some crucial differences between the smooth and polyhedral cases are described in the article of Cervone in this volume.)

Nico Kuiper is a model for so many of us. He left us a legacy of inspiring mathematical results, and even more importantly, a lasting love of mathematics that we can only hope to pass on to those who come after us, remembering our great friend as we do so. With affectionate gratitude we dedicate this volume to his memory.





With wife Agnete and Tom Banchoff, at Brown University.