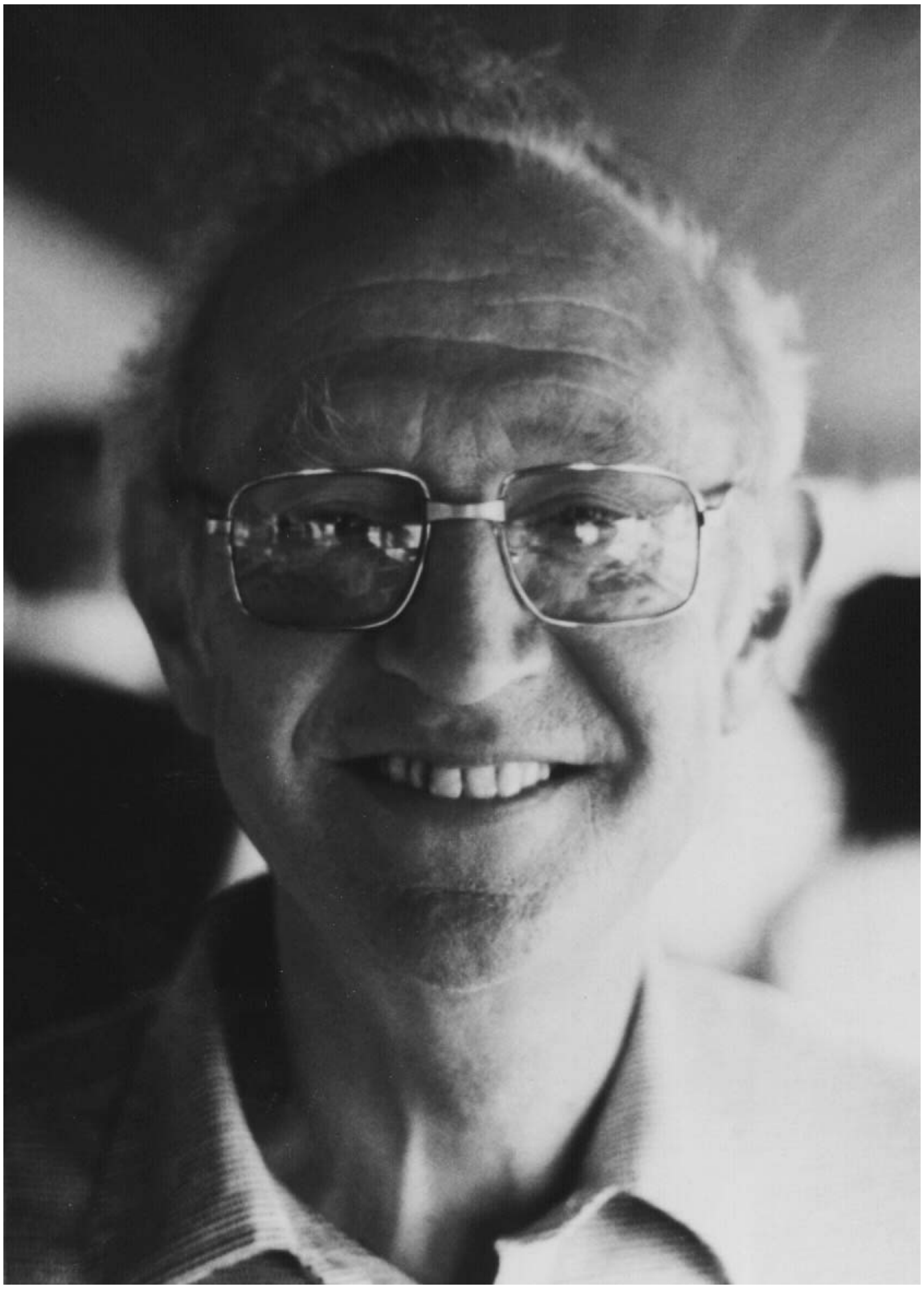

Nicolaas Hendrik Kuiper, 1920–1994

Tight and Taut Submanifolds
MSRI Publications
Volume **32**, 1997

Preface

This book grew out of a series of talks and closely related papers on tight and taut submanifolds given at the Workshop on Differential Systems, Submanifolds, and Control Theory held at MSRI on March 1–4, 1994. The workshop was organized by Robert Bryant and Shiing-shen Chern.

The book is dedicated to the memory of Professor Nicolaas H. Kuiper, who died on December 12, 1994. Kuiper made major contributions to the field of tight and taut submanifolds over an extended period of time. In particular, his technique of the analysis of topsets became an essential tool in almost all work in the area of tight immersions and maps. The book begins with a short description of Kuiper's life and work, written by Thomas Banchoff. Six of the seven subsequent articles cover various parts of the broad field of tight and taut submanifolds. The book concludes with an extensive bibliography of the field compiled by Wolfgang Kühnel and Thomas Cecil, a list of Kuiper's publications, and one of his doctoral students.

The first paper in the collection is an unfinished manuscript written by Kuiper himself. The paper was intended to be a survey of the field, and it is based on the Roever Lectures in Geometry that Kuiper gave at Washington University in St. Louis during the period January 20–24, 1986. In its current state the article is a masterly introduction to the subject and a good exposition of some more advanced topics, concentrating on topological aspects, in particular the analysis of topsets. It also contains a detailed proof of Kuiper's remarkable result that a tight two-dimensional surface substantially immersed in $\mathbb{R}^5$ must be a Veronese surface. We have made a few editorial notes in the text to aid the reader at appropriate points.

The second paper in the book, by Thomas Banchoff and Wolfgang Kühnel, is a comprehensive survey of the smooth and polyhedral portions of the theory of tight immersions, including many open questions. The article is self-contained, and there is some overlap with Kuiper's, since both begin with the basic definitions and examples. However, the two works are written from different points of view, and the polyhedral case is given far more emphasis in the second. Although many aspects of the smooth and polyhedral theories are similar, there are also points of significant divergence between the two theories. Banchoff and Kühnel pay particular attention to these points of contrast.

An important special case of the difference between the smooth and polyhedral theories is the subject of the third paper, by Davide Cervone. In a paper published in 1992, François Haab resolved a problem posed by Kuiper in the early 1960's by proving that there does not exist a tight smooth immersion of the real projective plane with one handle ($\chi = -1$) into $\mathbb{R}^3$. Surprisingly, Cervone produced a tight polyhedral immersion of the same surface into $\mathbb{R}^3$. Here Cervone describes his example in detail and provides a careful analysis of the difference between the smooth and polyhedral theories in this important case.

The fourth paper in the collection, by Thomas Cecil, is a survey of the closely related notions of taut and Dupin submanifolds in Euclidean space. This is a rich theory with many beautiful examples from the theory of isoparametric and homogeneous submanifolds. The relationship between the tautness and the Dupin condition is discussed thoroughly. There are both local and global aspects to the subject. Most local results have been obtained in the context of Lie sphere geometry, and this approach is described in the article in some detail.

In the next article, Chuu-Lian Terng and Gudlaugur Thorbergsson use the critical point theory of Raoul Bott and Hans Samelson to extend the notion of tautness to submanifolds of arbitrary complete Riemannian manifolds. They obtain several new classification results in this more general context. This far-reaching paper opens up many new avenues for research. It is followed by a short paper by Daniel Ruberman, where the author proves a topological result needed by Terng and Thorbergsson about null-homotopic embedded spheres of codimension one.

The final paper in the collection, by Ross Niebergall and Patrick Ryan, is a survey of results on real hypersurfaces in complex space forms with special curvature properties. This field has developed extensively over the past twenty years, and the authors provide a cohesive context for a wide range of results, leading to the frontiers of current research. Particular attention is given to Hopf hypersurfaces and hypersurfaces with constant principal curvatures. These are clearly related to isoparametric hypersurfaces in spheres, which play a prominent role in the theory of taut and Dupin submanifolds.

We wish to thank the authors for their contributions and for their help with various other aspects of the book. In particular, we appreciate the assistance of Thomas Banchoff and Wolfgang Kühnel in preparing Kuiper's article and the lists of his publications and doctoral students. We wish to thank Christine Heinitz, who prepared many of the figures for Kuiper's paper, and Davide Cervone, who did the same for Banchoff and Kühnel's. We also thank Silvio Levy, editor of the MSRI book series, for his assistance in preparing the book for publication, and Carol Oliveira for her help in typing Kuiper's manuscript.

Thomas E. Cecil
Shiing-shen Chern
Fall 1996