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Keynote Address: Mathematicians as Educators

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The mathematical sciences professions are in a phase transition, from which they may well emerge smaller, and/or redistributed and much more dispersed. We are not an endangered species, but our health depends on being able to transcend our historic tendencies toward insularity and on our outreach to all of our sister and client communities. This message, in diverse forms, is widely heard today.

The internal mathematical culture continues its deep investigation of the fundamental structures of number, space, dynamics, now with the added exploratory and processing power of new technology. These investigations are guided partly by purely intellectual evolution, but largely also by the natural sciences, to which mathematics furnishes the language and concepts for description, analysis, and modeling. In addition, mathematics provides design and simulation tools for engineering, technology, and for the organization and decision processes of industry. These diverse functions of mathematical thinking and tools are increasingly manifest in many professions, and across the technical workforce.

The phase transition mentioned above involves many partial shifts of focus — from core mathematics toward applications and toward interdisciplinary work with the natural and social sciences, from academic to industrial and laboratory settings, from individual self-directed work to collaborative and multidisciplinary effort, from technical communication with co-specialists to translational communication across disciplinary and cultural boundaries.

Mathematics education is meant to provide appropriate mathematical knowledge, understanding, and skills to diverse student populations. At the post secondary level, such education is entrusted to two large communities. One is based in our system of two-year and community colleges. The other consists of academic mathematical scientists, most of whom have been trained principally to do mathematical research, but for whom the economic base of their profession

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is now predominantly this educational mission. There has also been a small but distinguished group of scholars doing research and curriculum development in post-secondary mathematics, in the tradition of Pólya — for example Ed Dubinsky, Joan Ferrini-Mundy, Steve Monk, and Alan Schoenfeld.

The shifts described above are reflected in corresponding profound changes in the role of mathematics education. In the post WWII years we had designed a powerful educational model for producing an elite cadre of highly trained and motivated students destined for sophisticated scientific and technical careers. Some very able and committed mathematicians turned their professional energy to this educational task, often with inspiring success. But for the most part the pedagogy was formal, didactic, sometimes brilliant, and often severe. The many students whom it alienated and whom it filtered out of advanced mathematical study were deemed to fail the high standards of our calling. They were seen as lacking "the right stuff". Since the country did not require vast numbers of mathematically trained professionals, and there was sufficient mathematical talent and motivation to survive any pedagogy, this filtering system was considered benign. Many even considered it desirable.

The emergence of a highly competitive and technological world economy has fundamentally enlarged the demands on mathematics education. We now seek, for the broad workforce, levels of scientific and technical competence and literacy that approach what was formerly deemed appropriate for only a select and specialized student population. These same changes make increased demands of technical literacy for responsible and informed participation in our modern democratic society. These pressures give an added practical edge to the traditional argument for the cultural enrichment and intellectual empowerment that mathematical ideas and thinking can confer. When large numbers of students fail and/or leave mathematical study, which is the gateway to such competence and literacy, this is judged now to be the failure — not of the students — but of the educational system. Moreover, the students lost are disproportionately from the minority and female populations that constitute the major influx into the workforce.

The time has come for mathematical scientists to reconsider their role as educators. We constitute a profession that prides itself on professionalism, on an ethos of quality performance and rigorous accountability. Yet academic mathematical scientists, who typically spend at least half of their professional lives teaching, receive virtually no professional preparation or development as educators, apart from the role models of their mentors. Imagine learning to sing arias simply attending operas; learning to cook by eating; learning to write by reading. Much of the art of teaching — the thinking, the dynamic observations and judgments of an accomplished teacher — is invisible to the outside observer. And, in any case, most academic mathematical scientists rarely have occasion to observe really good undergraduate teaching.

While one does not learn good cooking by eating, neither does one learn it just by reading cook books or listening to lectures. Cooking is best learned by cooking, with the mentorship of an accomplished cook; that is, by an apprenticeship model. In fact, teacher education also is designed with a mixture of didactic and apprenticeship instruction. Professional development of academic mathematical scientists as teachers should perhaps be similarly modeled on learning in the context of practice, with only relatively small doses of the more formalized styles of learning with which we are most familiar. Good designs for doing this in a systematic way are not now common. Education professionals can help us in creating and experimenting with such designs.

Effective teaching requires that a teacher know his/her students, to be able not only to explain things to them, but to be able to listen to them, closely, and with understanding. And knowing something for oneself, or for communication to an expert colleague, is not the same as knowing it for explanation to a student. Further, the experience of a mathematical scientist as a learner may not be the best model for the learning of his/her student. These are the kinds of skills and awarenesses that professional development can help cultivate.

Of course there have always been in our professional ranks some very effective, even inspiring, teachers. They have become so through a combination of talent, personal commitment, hard work and practice — and without recourse to professional educators. But do these isolated individuals constitute a model for the educational responsibility of our profession? Are we — and the public we serve — to be content with the condition that some few among our ranks have chosen to take the individual initiative to develop their teaching skills? Imagine, by analogy, abandoning our disciplined education in rigorous mathematics for future researchers to a *laissez faire* system of individual self-instruction, when the impulse happens to be present. How might that affect the quality of our research community?

The disposition of many mathematicians toward the problems of education well reflects their professional culture, which implicitly demeans the importance and substance of pedagogy. Mathematical scientists typically address educational issues exclusively in terms of subject matter content and technical skills, with the "solution" taking the form of new curriculum materials. Curriculum is, indeed, a crucial aspect of the problem, and one to which mathematically trained professionals have a great deal of value to offer. But taken alone it can, and often does, ignore issues of cognition and learning, of multiple strategies for active engagement of students with the mathematics, and of assessing their learning and understanding. Ironically, the mathematical preparation of school teachers is frequently entrusted to these same mathematical scientists, who are often neither trained in nor sensitive to the pedagogical aspects of teaching mathematics to young students. Pedagogy is not something to be added, after the fact, to content. Pedagogy and content are inextricably interwoven in effective teaching.

Pedagogy, like language itself, can either liberate or imprison ideas, inspire or suffocate constructive thinking.

In fact, change on this front has already begun to occur, most notably that stimulated by the so called calculus reform movement. (For an excellent report on this development, see "Assessing Calculus Reform Efforts" (ACRE), by Alan Tucker, MAA, 1995.) To the often skeptical mathematicians outside this activity, the phenomenon is seen as one producing new curricular materials, and of introducing much more systematic uses of technology in teaching calculus. These new materials have been the subject of animated and healthy debate, though some of the opponents have been so stridently and indiscriminately critical as to polarize discussion and impede rational discourse. On the other hand, the people actually engaged in reform calculus teaching typically have a different sense of its significance. They show the same healthy skepticism toward curriculum materials that mathematical scientists have always shown, and they exercise appropriate professional judgment on the manner and extent of use of these materials. What they find most significant about the reform is their personal transformation and the change in their professional practice as teachers. They gain a sense of having become members of a community for which the practice of teaching has become a part of professional consciousness and collegial communication, not unlike their professional practice of mathematics itself. It is the creation of this substantial community of professional mathematician-educators that is, to my mind, the most significant (and perhaps least anticipated) product of the calculus reform movement. This is an achievement of which our community can be justly proud, and which deserves to be nurtured and enhanced. In addition to the ACRE report cited above, the JPBM study on Rewards and Recognition in the Mathematical Sciences is an important gesture in this direction, one that is widely appreciated and cited by our colleagues in other disciplines.

Some might be inclined to cite the calculus reform movement as a case of teaching improvement without the aid of professional educators. On the contrary, there were instances of significant consultation with education specialists. Moreover, the mathematicians who were fully engaged in calculus reform from the early stages, and who had to design programs to prepare the teaching staffs for these new courses, effectively became education specialists with a particular kind of professional expertise. They were funded, and devoted a major part of their time to this development. (I do not rule out the possibility that an education professional may also be a mathematician.) Furthermore, it is quite evident that the pedagogical philosophy that guided the calculus reform powerfully reflected that expressed in the K–12 reform efforts, which came from the thinking of the professional education community.

Once persuaded of the need for improved professional development as teachers, as many mathematical scientists and/or departments (often under external pressure) have become, how do they go about achieving these new goals? How, without prior professional development as teachers, can we mathematical scientists design courses and/or programs to provide this now, for present and future faculty? Part of the answer is that we cannot do it alone, either as mathematical scientists isolated from experienced professional educators (who may themselves also be mathematically trained), or as individual mathematical scientists without the collective support of our ambient departments and institutional environments. Many mathematical scientists have tended to look upon education professionals with doubts bordering on ill-disguised contempt; it is not an easy proposition that we now have much to learn from them, and need their professional help. Much remains to be done to establish contexts for respectful communication and professional collaboration between mathematical scientists and education professionals — from school teachers to people doing education research. This is ultimately a two-way street, along which mathematical scientists can contribute to the disciplinary strengthening of school programs and teaching practice, while the teacher and education research communities can elevate the pedagogical consciousness and competence of academic mathematical scientists.

Mathematics education, unlike mathematics itself, is not an exact science; it is much more empirical, and inherently multidisciplinary. Its aims are not intellectual closure, but helping other human beings, with all of the uncertainty and tentativeness that that entails. It is a social science, with its own standards for evidence, methods of argumentation and theory building, professional discourse, etc. It has an established research base, from which a great deal has been learned in the past few decades; this new pool of knowledge has an important bearing on the educational performance for which academic mathematicians are responsible.

What kinds of things need to be done? At the very least, our graduate students, who regularly perform as TA's or instructors, must be given serious teaching preparation; not only for their duties while graduate students, but also for their roles as possible future university or college faculty, or even as school teachers. Even if their career paths do not take them into the academic world, much of what they need to learn in the way of teaching skills forms part of the broader need for better communications skills in diverse settings. Skill at speaking, instructing, and interacting will make our students better and more effective spokespersons in their work and communities for the importance of sound mathematics education. These skills will also make them more professionally versatile, and more effective in their work. Indeed, such professional development is appropriate for current mathematics faculty as well as for graduate students. In addition, mathematics education provides an important option in the design of new Professional Masters Degree programs in mathematical sciences departments. The resources that support such programs should also provide for ongoing professional educational development of current faculty.

A further important challenge is the design, by mathematical scientists in collaboration with education professionals, of mathematics courses, based in mathematics departments, and devoted to the mathematical preparation of future school teachers. Of course one must distinguish here the needs of elementary teachers from those of secondary teachers. Teacher preparation is an area desperately in need of thoughtful development and experimentation, and which has not received the quality attention by mathematical scientists that it deserves. It invites the possibility of some novel and creative collaborations, where conventional ways of thinking have repeatedly failed to produce desired results.

The above kinds of efforts can be greatly facilitated by networking with colleagues on other campuses at which similar efforts are more highly evolved. There are various activities organized by the Mathematics Education Reform (MER) network and in special sessions at the winter joint meeting of MAA/AMS that support such networking.

While mathematics and mathematics education in the US at the school, college and graduate levels have historically been culturally and professionally separated — a separation visible in the distinct agendas and cultures of the AMS, MAA, AMATYC and NCTM — it becomes clear to anyone who contemplates the needs for improvement of mathematics education in America that this problem cannot be realistically segmented into components for which these four communities take separate and uncoordinated responsibility. As mathematical scientists, as mathematics education researchers, and as teachers in universities, colleges, community colleges and schools, we must begin to see our concerns for graduate, undergraduate and K–12 education as parts of an integrated educational enterprise, in which we have to learn to communicate and collaborate across cultural, disciplinary, and institutional borders, just as we are called upon to do in mathematical sciences research.

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