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Preface

In the collective unconscious of most working mathematicians, there is the lambent vision of a mathematics department in which mathematicians sit quietly in their offices and prove theorems. Occasionally they are interrupted by a duty to go teach, or to serve on a committee. But, for the most part, they think about mathematics. The mathematics department in this vision has a truce with the University administration: all the mathematicians have research grants, are regularly invited to other universities to give colloquia and to consult, and are recognized scholars; as a result, the administration does not press too hard about the quality of teaching, or how well the department serves the needs of students from other departments in science and engineering.

For better or for worse, the mathematics department described in the last paragraph no longer exists. There are several reasons for this change. Forty years ago, when that chimeric mathematics department *did* exist, it could safely be said that mathematics was an elitist subject. Society's demand for the mathematically fluent was quite small, and those few who survived the mathematics curriculum were more than sufficient to fill the need for teachers, researchers, and mathematical scientists. There was no need for self-examination because what we did seemed to work.

There is now a broad perception that what we do does *not* work. Society now demands a technologically literate work force, and the elitist teaching methodology developed by earlier generations of mathematicians is no longer adequate to the job. More precisely, it is no longer adequate in view of the broad cross-section of society that we are trying to educate, and in view of the level of technical competence that is demanded of those whom we graduate.

Put a different way, we mathematicians must learn to be consciously aware that most of the students in most of our classes are not future mathematicians. With few exceptions, they will not be going on for graduate work in mathematics; and in many cases they may never take an upper division (or a rigorous) mathematics course. We need to understand and embrace the observed fact that a business student has a real need to understand the meaning of the statement "the rate of decrease of inflation is increasing"; this student probably does *not* have any use for knowing how a predator-prey problem can be modeled using a pair of coupled ordinary differential equations.

Forty years ago the observation that 60% or more of freshman fail calculus would have been considered confirmation of the rigors of our curriculum. Now the same statistic (this statistic, and worse, is valid country-wide) signals that something is fundamentally wrong with the way that we teach calculus, and perhaps with the way that we teach undergraduate mathematics overall.

In 1986, Ronald Douglas convened a small conference at Tulane University to examine the state of modern mathematics teaching. By way of this meeting, the "reform" movement in mathematics teaching was born. Although "reform" has a different taste in different people's mouths, it is safe to say that the characteristics of reform are an emphasis on concepts rather than calculations, a creative use of technology, and a stress on geometric insights. Reform teachers use group work, Socratic dialogue, discovery, computers, and other non-traditional methods to get students engaged in the learning process.

Not all mathematics instructors are receptive to the tenets of reform. Many who have taught for decades using lectures, and who feel that their lectures have been effective, are resistive to "throwing out the baby with the bath water" as they are pushed to embrace the new techniques. Debate over the merits of reform, and how it should be implemented, has sometimes been quite heated. In the ensuing discussions, invective has replaced careful reasoning, intuition and conjecture have replace fact and study, and our hard-won scholarly method has often been forgotten.

While many "calculus reform" projects have prospered with the aid of federal grants, it has been convenient for most mathematicians at research universities to ignore the studies and results of the reform movement. Those mathematicians prefer to live in the dream mathematics department described in the first paragraph, and to treat the reform movement as background noise.

The present book is the outcome of an effort to create a dialogue about mathematics education that went beyond calculus reform, and that included mathematicians with a wide variety of views. In a conference held at the Mathematical Sciences Research Institute in Berkeley on December 5 and 6, 1996, with support from MSRI and the National Science Foundation, more than one hundred mathematics instructors, all dedicated to effective mathematics teaching but by different means, engaged in meaningful and sometimes spirited discussions about how mathematics can and should be taught. The conference consisted of formal presentations, question and answer sessions, and more informal "work groups". Our volume contains reports and position papers stemming from all these activities.

As the reader may imagine, the participants in our conference were anxious to be open-minded and to engage in a civilized give-and-take with participants of all stripes. But there was some animated debate, and even some heated dialogue. One of the most interesting interchanges concerned the issue of how "non-proofs" should be presented to a calculus class. For example, if you state the Fundamental Theorem of Calculus and explain why it is true with the usual picture depicting the area under the function, from x to $x + \Delta x$, approximated by the area of a rectangle, then should you take pains to tell the students that this really is not a proof — it is in fact a heuristic? Some traditionalists strongly favored the notion that proofs are sacred, and when you show a class something that is not strictly a proof then you should say so. Others, including some reformists, felt that if you say "here is an idea of why this is true" or "this picture will help you see why this is true," you will have covered all bases and will not have lied. Of course, there is no one correct answer to this issue, and there is much room for disagreement.

The methods that many of us have adopted in our teaching have primarily evolved through repeated experience and through trial-and-error. Few of us have ever had any formal instruction at teaching, and few of us have ever engaged in any formal discussion of issues of pedantry. Thus, for many participants in this conference, there was the joy of discovery of a new sort of discourse. There was also the joy of discovery of new and untapped emotions.

The pedantic issue (about proofs) raised in the last paragraph but one was never settled at our conference. It is safe to say that the main accomplishment of the meeting, apart from giving people the opportunity to make new acquaintances and engage in networking, was to sensitize everyone to a number of important teaching issues and techniques.

And that is really what showing people how to teach is all about: it is decidedly *not* to inculcate in them any particular set of values, nor a particular teaching methodology. Rather, it is to acquaint them with the goals of teaching, the problems that may arise, and with various methodologies that one might use to handle them.

The essays in this volume address the new teaching environment in which we live and work, the newly structured society that we serve, and the new sets of goals and values that are being set for every mathematics department in the country. We hope that they will be of value to everyone who is striving to be an effective teacher. And we also hope that they will be the basis for further productive discussions of teaching issues.

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