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The Mathematics Major at Research Universities

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In the past decade the focus in university science and mathematics education has shifted from providing an adequate supply of world-class professional scientists to the broader agenda of providing excellent education in science and mathematics to all undergraduate students. In the words of David Goodstein, "... the United States has, simultaneously and paradoxically, both the best scientists and the most scientifically illiterate young people: America's educational system is designed to produce precisely that result. America leads the world in science — and yet 95 percent of the American public is scientifically illiterate." In an environment where jobs that provide decent economic opportunity demand skills far more sophisticated than those required in the past, universities are now being called upon to provide all of their students both with a supportive environment for acquiring these skills and with the ability to continue learning throughout a lifetime in the workforce.

Many of the current educational changes are driven by this new agenda. Major efforts have been and are being made to improve learning, primarily at the freshman-sophomore level, for students who are not planning to become mathematicians, research scientists or engineers. At the same time there is great concern that the parts of our system that have led to the successes of American science and technology not be destroyed. Indeed, a significant part of the current debate on calculus reform (a term much disliked by parties on all sides of the issue) concerns how best to achieve both of these goals. In the current environment, mathematics departments are under pressure to succeed at both. Further, we must also integrate into our courses the use of powerful computational tools and somehow do all of this without increasing either course credit or the amount of time students spend on our classes.

The educational programs in mathematics at large research universities such as ours, the University of Michigan, divide naturally into three pieces according to the interests of the majority of the students at that level: the freshman-sophomore program, the junior-senior-masters program, and the doctoral program. The doctoral program (and the small undergraduate honors program) receives a lot of faculty attention, primarily because the courses and research experiences given to its students are very close to faculty interests. There is no doubt that this attention is deserved because this high-value program produces the future mathematics researchers and university teachers. Freshman-sophomore programs also receive a lot of attention from departments because of their size and because they "pay the bills." At Michigan, 80% of our departmental teaching load is in the freshman-sophomore courses and 97% of the students in these courses are not mathematics majors. Furthermore, while research is a very important part of faculty work, revenues derived from external sources for research support account for a small percentage of the financial support of mathematics departments; most resources come to us because of our role in teaching mathematics. Of course, the freshman-sophomore programs are also important educationally because of the role of mathematics as an enabling discipline for all of science and engineering, in addition to many other subjects.

Our focus in this article is on the intermediate level, the program for math majors and advanced students from other disciplines. In particular, it seems to us that, especially at large universities, the undergraduate mathematics major who is not heading toward graduate school is the forgotten student. These are the majority of our math majors: at Michigan about 10% go on to graduate school in mathematics and another 30% go to graduate school in another discipline, e.g., law, medicine, industrial engineering, biostatistics, etc.; this leaves 60% who go directly into the workforce. Of these, roughly one-fifth go into K–12 teaching.

In particular, if we are to have workers with high levels of technical skills in the major industrial sectors of our economy, including computing and information technology, banking, insurance, and communications, then many of these positions should be filled by mathematics majors. Indeed, that is where many of our current majors go. We consistently hear from our alumni in business that the level of technical skills needed in many occupations is increasing. As one example, various kinds of contingent securities (or"derivatives") are now commonly and increasingly used in most large businesses. While we may produce enough "rocket scientists" to design and price such financial instruments, do we have enough accountants with sufficient technical background to quantify and independently evaluate their risks? When problems arise, lawyers involved in litigation need a good understanding of the technical aspects of their cases. The mathematics major is a natural arena where students can acquire the needed skills. We should view it as an important part of our mission to encourage more young people to become math majors and to help them obtain in addition to mathematical training the broad liberal arts background that helps them succeed in such careers.

Adopting this point of view raises several difficult and important questions. What exactly are the needed technical skills? What exactly is the mathematics we should teach? Do our current programs do a good job of teaching these skills and topics? In what ways can the special resources of research universities be brought to bear on the problem? How can we communicate to our undergraduates the value of majoring in mathematics?

Except for basic computer skills (programming, use of spreadsheets and word processing software) particular technical skills do not seem to be high on the wish-lists of prospective employers. Rather, the ability areas most often mentioned to us by alumni in the field are problem-solving, breaking complex problems into solvable pieces, adapting a solution from one problem setting to another, writing and otherwise communicating with clients and co-workers, decision making, working well with others in a team, and a willingness to work hard. Many of these are standard components of an undergraduate mathematics curriculum. Much of mathematics deals with solving problems, and usually requires breaking them down into smaller pieces. The abstraction of mathematics is exactly designed to extract principles common to many contexts. Probability and statistics deal with making decisions in the face of uncertainty. Courses in mathematical modeling, which our students almost universally recognize as valuable, deal with choosing the right tool to solve a problem.

Thus it seems that the basic content of our undergraduate courses is appropriate to these new, or at least newly recognized, goals of the undergraduate program. What may be less appropriate or adequate is our pedagogy. Most of us who are mathematics faculty learned throughout our education, but particularly in graduate school, that mathematics is a solitary occupation. We were probably good at timed exams, and when we didn't understand something, we would think and read much more before we would ask for help. These are good approaches for research mathematics, but they are not the way a B.A. mathematician works in industry. In all of the many recent conversations we have had with our alumni in the workforce, we have not found a single example of a work situation that resembles a typical course exam or a problem that has a clean, unique solution in the style of the standard textbook exercise. All of these alumni stress almost as a mantra the importance of teamwork and communication — indeed, these abilities are rated above the raw technical knowledge that we are constantly struggling to impart.

A common element of calculus reform projects has been the introduction of cooperative learning. In the first-year courses at Michigan most submitted homework is done in groups, and much lecturing is replaced by group exercises in the classroom. It is surely time that some of these strategies begin to be adopted also in the upper-level courses. Of course, there will and should remain a good deal of individual work, but we are convinced that we should continue to strengthen students' ability to solve problems cooperatively throughout their undergraduate curriculum. Changes in the first- and second-year courses have also increased the emphasis on communication; gone are the days when a scrawled formula or number counted as a solution to a problem. In most instances in these courses we now require a coherent explanation of the solution in full, good, English sentences. But again, this change has only begun to percolate through the upper-level courses. It is not too much to ask that every course should include at least a small requirement of careful written work.

These changes in undergraduate pedagogy are not easy to make. Faculty of our generation are experienced and comfortable with the lecture model and often at first very uncomfortable with supervising collaborative groups. Teaching communication and writing is difficult and expensive; grading homework and exams with attention to both writing and content is much more time consuming. We will not be able to do it by fiat, but it should be a long-term goal. Furthermore, there are many opportunities beyond the regular classroom to reinforce these lessons. Student research projects, ideally under the auspices of an REU (Research Experiences for Undergraduates) or similar program, are ideal for encouraging both collaborative work and careful written exposition of the results. Although traditional paper-grading jobs are useful for solidifying technical mathematical knowledge, jobs as tutors or classroom aids are far richer in providing experience in communicating mathematics in a cooperative setting.

Our math major program may not be doing a good job of training high-school teachers, judging from what we see of entering university students and what we read in the news. What can research departments do to improve the situation? Probably very little, since it is a large, national problem, beset with local politics and other human issues. At Michigan we have only a small program in this area, and these authors have little experience and no suggestions in this direction. However, the improvement of mathematics instruction and standards in K–12 education is clearly an important problem which should be of concern for mathematicians in research universities.

Another place where we can improve is in the counseling of our majors. The mathematics curriculum is not the only or even the best place to work on developing all of the skills our graduates need; there are many more opportunities for learning than any one department can provide. Mathematics courses take up no more than 30–40% of the junior-senior program for mathematics majors at Michigan. Guidance in the form of suggested programs should be given for many more, perhaps another 40% of their courses. For example, students interested in careers in business, insurance, banking, or information technology should be advised to take courses in economics, computing, writing and speaking, accounting, etc. Students interested in engineering or computing areas should take more courses in computer science, such as databases or operating systems, operations research, modeling, physics, biology, etc. Most students do not have clear ideas of what they want from a program, or which courses to take, and will appreciate having clear recommendations. Furthermore, few employers of bachelors degree students are looking for graduates with specific mathematical skills. For most students, having a broad education that cuts across many disciplines enhances their job prospects. This doesn't mean having a hodge-podge of random courses on the transcript, but taking a broad range of courses that give a strong knowledge base of complementary skills relevant to a general career direction.

In parallel with efforts to improve the curriculum for the undergraduate math major, we need to address the problem of recruitment. How can we show the sophomore deciding on a major that mathematics is a good choice? One obstacle is that mathematics is hard — there is no way our typical major can learn mathematics without serious study. On the other hand, hard work is a characteristic of every scientific field, and there is no reason to believe that students are afraid of hard work. Indeed, a strong work ethic has a lot to do with career success in any field. We should not weaken our programs or relax standards with the idea that this will attract majors. Rather, our students should be made to work hard and to understand that their efforts will be rewarded. We should make every attempt to find out how our students learn best, encourage them in their efforts, and offer them every opportunity to succeed.

Another obstacle to recruiting efforts is that beginning students, as a group, have little idea of what math majors do other than become teachers. We are handicapped by the fact that there are almost no positions below the Ph.D. level with job title *mathematician*, while titles such as *engineer*, *lawyer*, and *economist* are part of everyone's experience. Furthermore, many mathematics faculty don't themselves have a much better answer to the pervasive question "what can I do with a math major"? Fortunately, the national mathematics organizations, most notably the MAA, have produced several excellent brochures and web pages (some are accessible from the Michigan Department's Web page under Student Resources) with examples of the very wide range of opportunities available to the mathematics major. Furthermore, we have found that alumni are willing and often eager to return to their *alma mater* to tell students of the possibilities in their fields.

Faculty need to become better aware of these career opportunities. Once we do, we have an obvious medium for proselytizing: each year we teach calculus to several hundred thousand students. Current texts often treat many interesting applications of mathematics that show its utility in other disciplines. But do the texts and mathematics faculty point out when the occasion arises that these represent possible career paths for mathematics students? For example, one commonly given application of the integral is to compute present or future values of money. Why not take this opportunity to mention careers in actuarial science, banking, and finance? Optimization problems could be linked to careers in communications and transportation.

Throughout all of these efforts to provide a better undergraduate environment for all of our students, we should not lose sight of whom we are serving. Although our students will be the leading citizens of the next generation, most of them will not be the creative wizards who achieve breakthroughs in their fields. They will take the tools that we give them and use them in ways we might not recognize but which are nevertheless crucial to the evolution of society in the information age. We should not expect them all to perform at the level of honors students, but recognize that what will be important for them is as much the overall structure of mathematical thinking as it is the detailed content of our courses.

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