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If I Could Talk to the Animals

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The title of this essay is a free-floating reference to conversations I have had with my mathematical colleagues over the last few years about the extent to which the classes we teach within our departments ought to inform and connect with the subjects students are studying in other departments. These discussions often end with a complaint such as, "You can't talk to our engineers, they won't listen." or "The biologists just don't care about the math their students take." or "How can you expect an English teacher to have an idea what kinds of math students ought to know?" I'm not sure whether such comments were made with the specific intention of ending an uncomfortable conversation, but they certainly had that effect. I would also be willing to bet that conversations with the engineers, the biologists and the English professors had stopped sometime earlier, if indeed they had ever happened in the first place.

We are at a critical place in the history of mathematics education. Mathematical thinking has insinuated itself into every scientific discipline, including social science. Science and technology in turn have affected every aspect of human society, including art, literature and music. It would be a particularly inappropriate moment for the community of mathematicians to turn inward upon itself for answers to questions concerning the education of students in all disciplines. We must seek these answers jointly with scholars in every area. Only in this way can we construct mathematics courses meaningful to students with a variety of interests and inclinations. Dartmouth was recently awarded a grant from the National Science Foundation under its initiative, "Mathematics and its Applications Throughout the Curriculum". Because of this work, which is expected to result in the creation or reworking of between twenty and thirty courses in our curriculum, some of us have been having conversations with colleagues all over campus about how various students might benefit from the mathematics they take here. As you will see, the answers and suggestions we have been hearing are creative and often surprising. My intention here is to describe some of the conversations I have had with faculty throughout my college; I think you will be surprised by some of the unexpected answers I got and impressed with the willingness of others to seriously consider this issue. I hope it will motivate you to open the way for similar conversations at your institution.

Like many math departments, for years mine had been attempting to have conversations with the physicists and the engineers, whose need for mathematical preparation of their students was very clear cut. General hostility was the usual outcome. The mathematicians felt that the demand for superficial coverage of topics was considerable; as a result, someone would have to invent a pill for students to take, prior to their arrival at college, which mysteriously deposited a repertoire of familiarity with special functions of every type. This seems to be what would satisfy the professors of engineering. The engineers seemed mystified with the mathematicians' seeming unwillingness to teach much mathematics. You would correctly guess that the "calculus reform" effort exacerbated this argument greatly. So, in the last couple of years, we have taken a different tack in our conversations with engineers and physicists. Instead of asking the question, "What topics should we cover?", we have instead asked these faculty to identify for us the specific ways in which their students fall short of expectations in their sophomore classes.

Initially the engineers said, "The students don't really know what a derivative is." We said, "We have tested them in every way possible and they seem to know it when they leave the calculus class." Further discussion yielded two points of agreement. First, the students do not retain material as well as we would like. Second, the students do not recognize concepts in a new context (which in this case was any context outside the calculus class). Once we agreed on these two problems as the focus of our efforts to improve calculus instruction for physical science students, all of these faculty were able to collaborate together to create a joint syllabus in mathematics, physics, and chemistry. The mathematicians argued for covering less material in order to achieve greater student retention. The physicists and engineers argued for an interdisciplinary approach that would improve the students' ability to transfer concepts and skills from one context to another.

Like many math departments, we had never really had a conversation with the biologists. In our cosmology we had pretty much equated biology students with pre-medical students, whose reputation for insincere interest in mathematics was made for them by their predecessors twenty years ago. The pre-med students had a similar reputation with biology professors. So we now make a distinction between the intellectual endeavor of biology and the presumed attitudes of the majority of inhabitants of poor biology's major. It was also surprising to learn of the level of interest among some biology professors, in particular ecologists and population biologists, in the mathematical education of their students. The biologists said, "We would love to have our students take more math after basic calculus, but (i) none of your courses is directly applicable to our needs. Furthermore, (ii) none of the biology students believe that calculus is in any way pertinent to biology. In fact, (iii) they don't even recognize a derivative when they see one in class."

This gave us three problems to solve, none of which is easily addressed. The last two problems can be directly addressed by an interdisciplinary approach to beginning calculus intended specifically for biology students; this would be similar to the solution agreed upon by the physicists and engineers in the last example. But the absence of courses specifically applicable to the needs of biology students requires entirely new courses. And what will drive the mathematical content of these courses? The biologists said, "All we want our students to be able to do is to understand what a phase portrait tells you and to have a good working understanding of what an eigenvalue of a matrix is." They further pointed out that most topics in the freshman courses, such as methods of integration and properties of special functions and vector calculus, were quite irrelevant to the biologist. Here you see directly reflected the needs of both population biology and ecology. In fact, what you are seeing is a change of emphasis in the fields which compose biology itself.

Oddly, even though mathematicians tend to see mathematics as generally useful, it is the mathematics department which must be convinced to serve the needs of the students in this case. Phase portraits and eigenvalues are topics generally reserved for advanced courses which serve primarily math majors. An adjustment of this sort reverses the somewhat proprietary stance we often take toward this material. We will no longer be able to say that the whole purpose of linear algebra lies in its ability to teach future mathematicians how to write proofs. Nor can we say that the purpose of dynamical systems is to generalize the classical approach to ordinary differential equations. The biologists were very clear about this. They said, "We never met a differential equation we could solve analytically in our research."

The biology professors seem to regard exponential growth and the logistic equation as curious anomalies that serve to lead to the more interesting questions. When asked how they approached eigenvalues they (somewhat sheepishly, I thought) said that they defined the eigenvector to be the limiting population distribution in their model. They weren't too happy about that approach either, but it was all they could do, given the background of their students. As a result of these conversations, the math department is now giving serious consideration to an alternative calculus sequence for biology students whose content would be quite different from the sequence an engineer or physicist would take.

Like many math departments, we rarely consider the needs of nonscience students. We have created several courses with no prerequisites; these are to serve those students desiring breadth of education and increased familiarity with mathematics. But I don't recall ever having a departmental discussion of why an art or literature student would want to take a math course, or how such a course could directly speak to their existing interests in art or literature. To me, this is one of the most intellectually engaging questions of its sort, precisely because it is so hard to answer and also because the answer could not possibly come from within the mathematics department alone.

Most of the time, when I ask this question of humanists, the answer I get is sheer nonverbal puzzlement. The question is as foreign to them as it is to us. Once in a while I get a big clue, though. For example, one person pointed out that the earliest science fiction was written during the Renaissance, and was obviously a direct response to some of the science of the time. Another pointed out that there were several modern authors dealing directly with mathematical and scientific issues in their works, and he wished that when he taught those works he could give the students the necessary insights into the mathematical and scientific ideas. Yet another is troubled by how unaware students seem to be of how math and science have informed musical ideas. Putting these discussions together, we can see several strands.

One pervasive theme is the idea that all of culture, whether it be literature, music, or math, is part of a big picture whose parts are all related. That big picture, and the quality of those relationships, is the natural object of study for humanists, especially scholars of literature and history. Such an inquiry is not possible if the contribution of science and mathematics is omitted. Second, there is the suspicion that in some ways the doing of mathematics is more like the doing of art or music than it is like the doing of science. On the individual level, the conception of a mathematical idea is much like the conception of an artistic idea. The commonality of human endeavor comes to the fore here. The sensation of behaving intellectually like a mathematician, even for a very brief period of time, acquires a value far greater than the value of the mathematics invented at that moment. Finally, there is a perceived benefit in the opportunity to interact with modern scientific and mathematical ideas. The faculty I spoke with in no way expressed concern with the ability of their students to compute compound interest, for example. They were much more interested in thinking about very modern issues such as relativity and chaos theory, because they were convinced that these things have a profound effect on how human beings think of their relationships to each other and to the universe.

We see a lot of articles and discussion right now on the subject of "quantitative literacy"; this phrase nearly causes me to break out in hives because of the Orwellian overtones I detect and also because of some of the texts I have seen produced in its name. The whole concept of "quantitative literacy" is being conjured by mathematicians as the answer to the question, "what should the average citizen know?". I hope I have made it clear in this essay that such a question is wrong-headed. The "average citizen" is not what anyone really aspires to be; it is not in the mission statement of my institution, and I would bet it isn't in that of your institution either. (Dartmouth College aspires to produce average citizens. Please send your check for a hundred thousand dollars to _____ .) If you rephrase the key question, however, to read "how can mathematical knowledge serve every person?", then you have certainly created a question that mathematicians by themselves cannot possibly answer. If you then open this conversation to scholars from other corners of your institution, it will soon become clear that there is a huge diversity of human beings who would all benefit from mathematics for completely different reasons.

To increase the intellectual basis in mathematics for the citizenry of the next millennium, drastic action is needed on our part. The first step of that action needs to be serious discussion among people in all disciplines about how math ought to inform, gain from, and interact with other disciplines. Subsequent action must evolve as a response to this dialogue among disciplines. I look forward to a new type of conference involving people from a broad variety of scientific and other disciplines. This conference would engage in serious consideration of the problems raised here, both for the benefit of all of us and as a visible proxy for a process that must necessarily involve us all. Perhaps then the broad sea of mathematics will begin to take shape as a series of pools drawing students of many different disciplines, according to their needs. We must regard the diversity of all comers not as an obstacle to progress, but as what it truly is: our strongest national resource. We must begin to see that one size most emphatically does not fit all. Only then can we build a curriculum that addresses the problem before us in more than a superficial way.

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