More Games of No Chance
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An Application of Mathematical Game Theory to Go Endgames: Some Width-Two-Entrance Rooms With and Without Kos

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Abstract. The author is part of a research group under the supervision of Professor Berlekamp. The group is studying the late-stage endgame of Go and has extended the theory of quasi-loopy games using the concept of a ko-master. This paper discusses some width-two-entrance rooms with and without kos that have arisen in this study.

1. Easy Examples without Kos

Figure 1.1 shows a simple width-two-entrance 5-room position. Though it is easily analyzed by mathematicians, it is difficult even for strong Go players to determine the best move. Black's two options from Figure 1.1 are shown in Figures 1.2 and 1.3, and White's two options are shown in Figures 1.4 and 1.5.

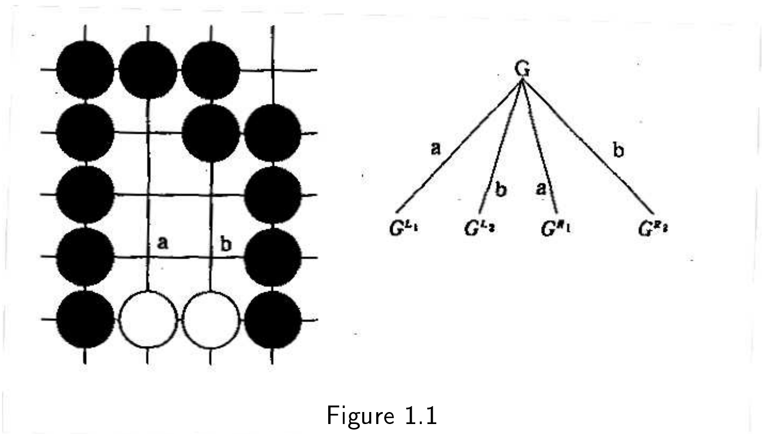


Figure 1.1

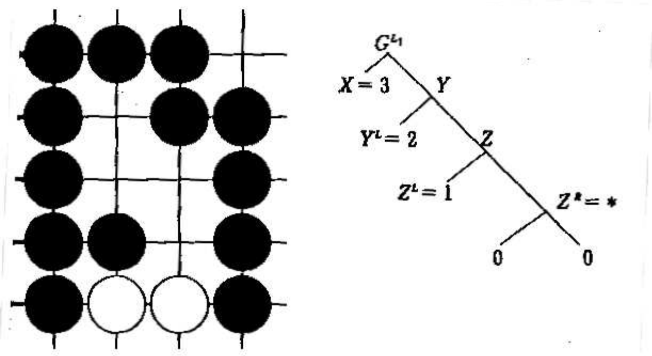


Figure 1.2

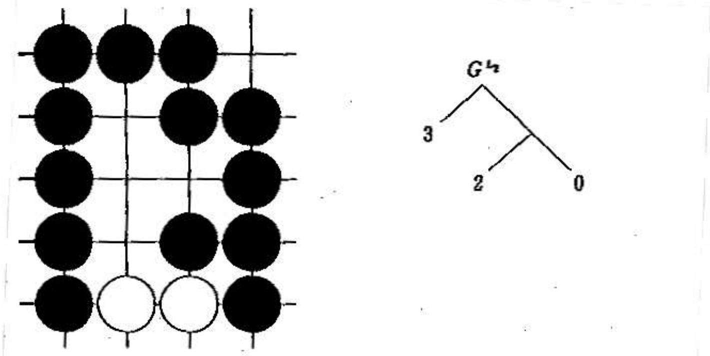


Figure 1.3

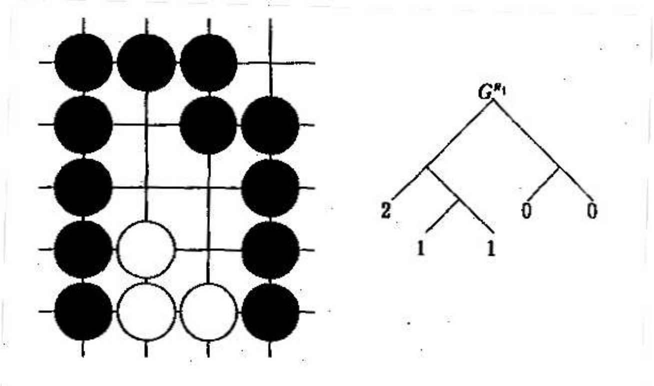


Figure 1.4

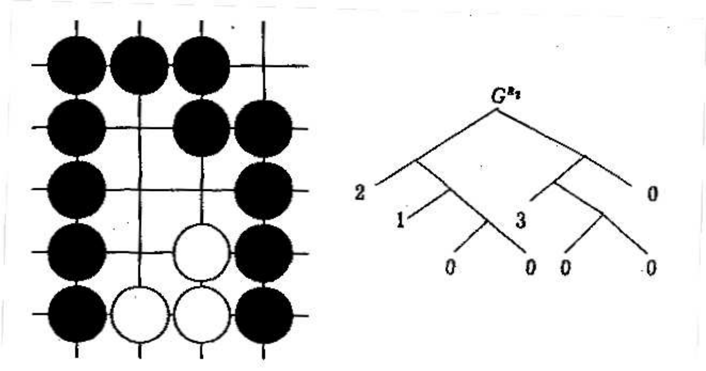


Figure 1.5

We get the following results: $G_1^{L_1} = 2 + \frac{1}{8}$, $G_1^{L_2} = 2\uparrow$, $G_1^{R_1} = \frac{3}{4}$ and $G_1^{R_2} = \frac{1}{2}$, thus $G^{L_1} > G^{L_2}$ and $G^{R_2} < G^{R_1}$. Therefore, Black should play a but White should play b from the position in Figure 1.1, and $G_1 = \{G_1^{L_1} - 1 | G_1^{R_2} + 1\} = 1 + \frac{1}{4}$ is the chilled game of G (Figure 1.6).

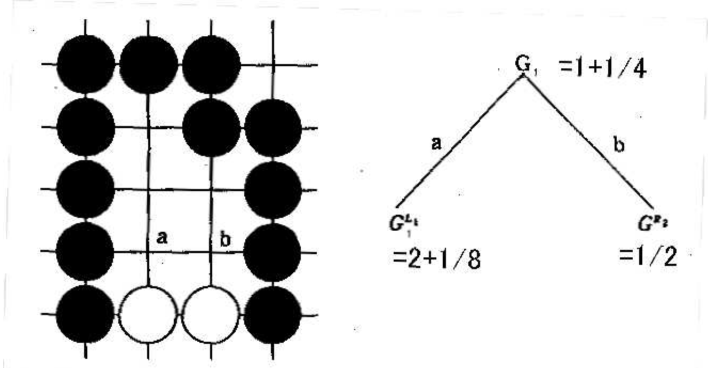


Figure 1.6

Figure 1.7 shows an even easier game than the game in Figure 1.1. Black and White should play in the same place, and these moves are analyzed in Figures 1.8 and 1.9. The canonical form of G and its analysis using the chilling method are shown in Figure 1.10. We get $G_1 = 2 + \frac{1}{8}$.

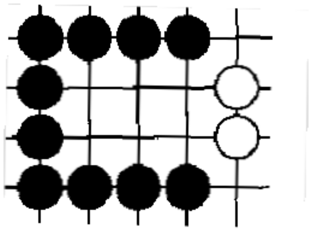


Figure 1.7

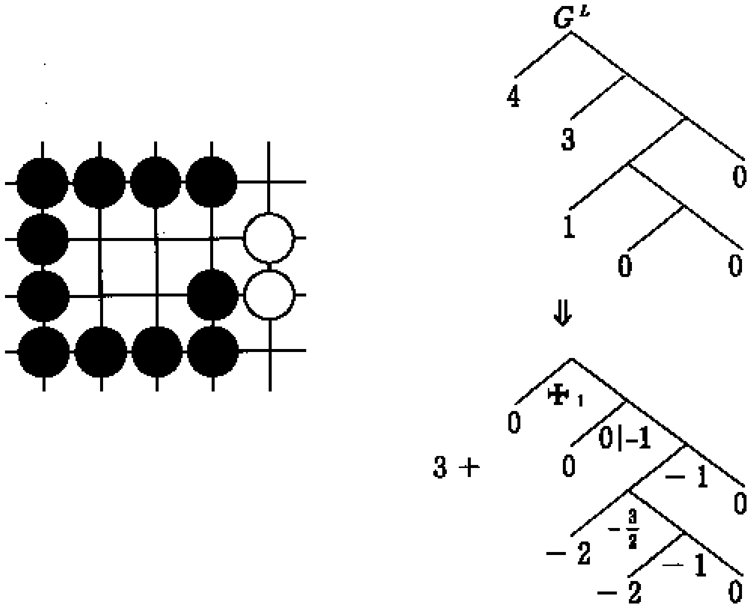


Figure 1.8

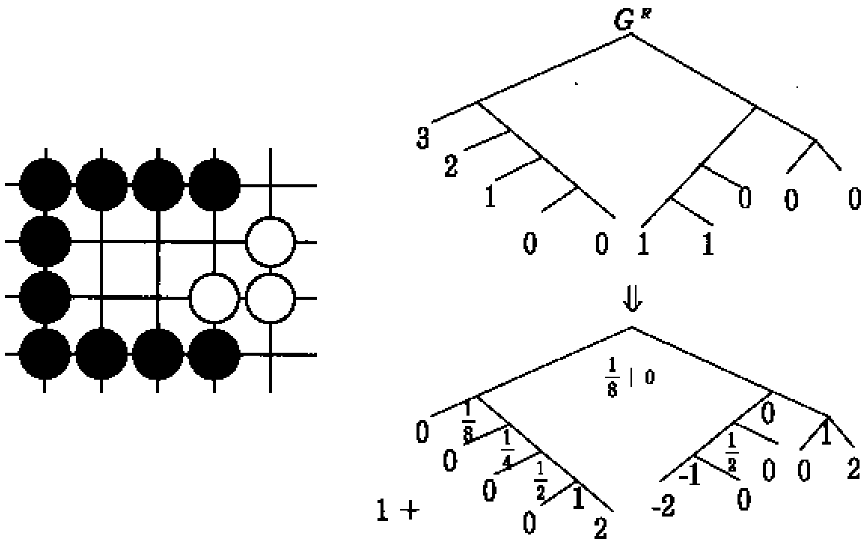


Figure 1.9

2. A Simple 1-point Ko

Figure 2.1 X shows a simple one-point ko. If Black plays A in X then the position moves to Y. If White can play B in Y then the position moves back to X. The ko rule bans each player from taking back a ko directly after another player takes it, so that a game position which includes a ko is not a loopy game but a quasi-loopy game.

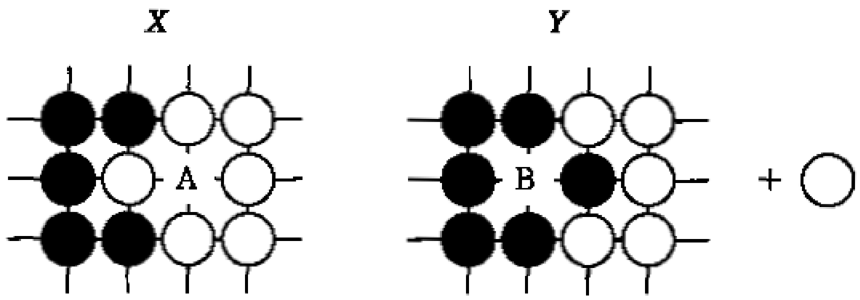


Figure 2.1

White has to play in some other place if Black takes the ko. If Black responds in the other place, then White may take back the ko. White's play in the other place is called a ko-threat.

Figure 2.2 (left) shows the game tree of X. It can also be expressed by a graph as in Figure 2.2 (right). Let game G be the game X in Figure 2.1. We can make three copies of X and name them X_1, X_2 and X_3. $X_1 + X_2 + X_3 = 3 \cdot G$ (Figure 2.3).

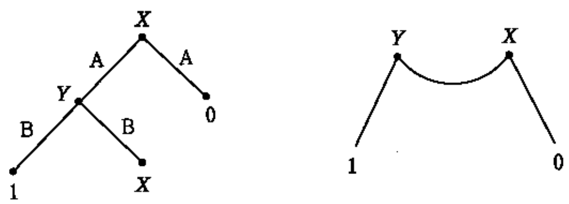


Figure 2.2

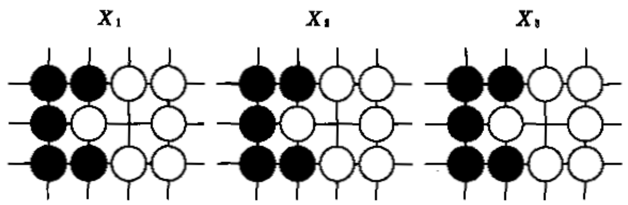


Figure 2.3

If Black plays first, he plays X_1 to $X_1^{\mathrm{L}}+X_2+X_3$. Because $(X_1^{\mathrm{L}}-1)+X_2=0$, White then plays X_3 to $X_1^{\mathrm{L}}+X_2+X_3^{\mathrm{R}}$ and the value of such a game is 1. If White plays first, she plays X_3 and then Black plays X_1 to reach the same position. Therefore, $3 \cdot G = 1*$. And the mean value of G is $\frac{1}{3}$.

When there is a ko, Black or White eventually fills the ko. The filling side is the winner of the ko and called the ko-master. Note that the ko-master must fill the ko. When Black is the ko-master, then the game G is denoted by $\hat{G}$; when White is the ko-master, then G is denoted by $\check{G}$.

Figure 2.4 shows the game graph where Black is the ko-master and Figure 2.5 where White is the ko-master. If White is the ko-master, Black may take the ko by playing at A in Figure 2.1, but when White takes back the ko, Black cannot take back the ko again. So the position is the same but the situation is different. The ko-master White should fill the ko, and we denote this position as $\dot{X}$ rather than $\check{X}$.

3. An Example Containing a Hidden Ko

Figure 3.1 shows an example of a Go position which contains a hidden ko. Black's options are shown in Figure 3.2, while White's options are shown in Figure 3.3.

Figures 3.4 through 3.6 show game tree analyses of chilled games:

$$\hat{G}_1^{\mathrm{L}_1} = 3 + \{0|\uparrow\} = 3 + \Uparrow *, \check{G}_1^{\mathrm{L}_1} = 3 + +_1, G_1^{\mathrm{L}_2} = 2 + \{1|*\}, G_1^{\mathrm{L}_3} = 2.$$

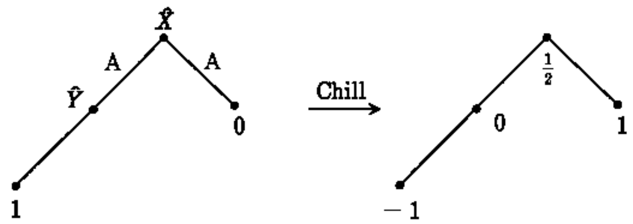


Figure 2.4

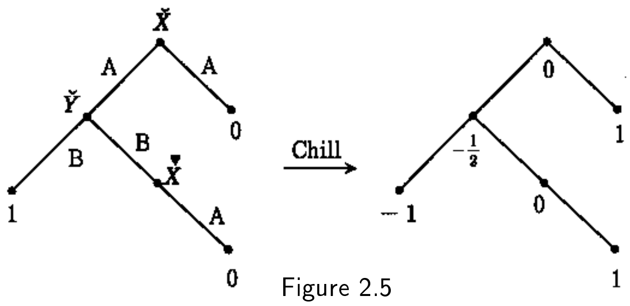


Figure 2.5

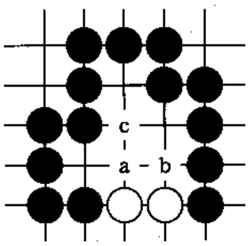


Figure 3.1

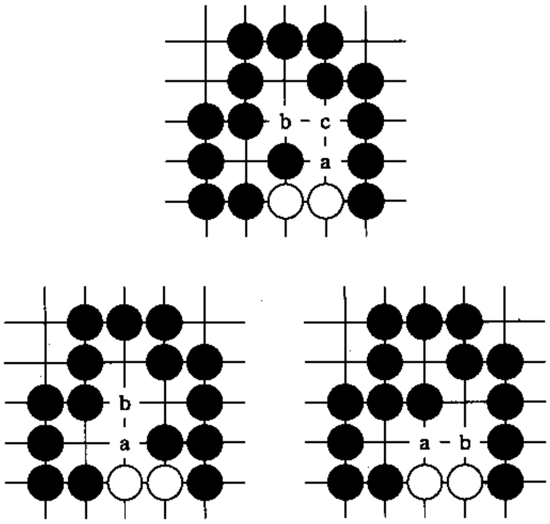


Figure 3.2

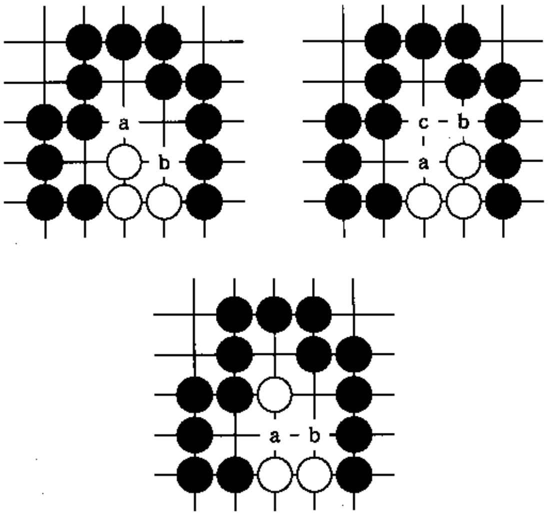
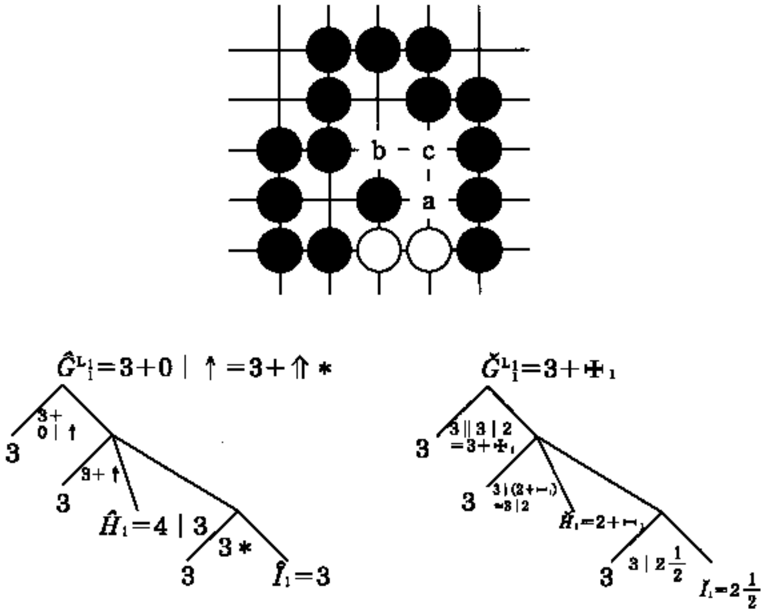


Figure 3.3



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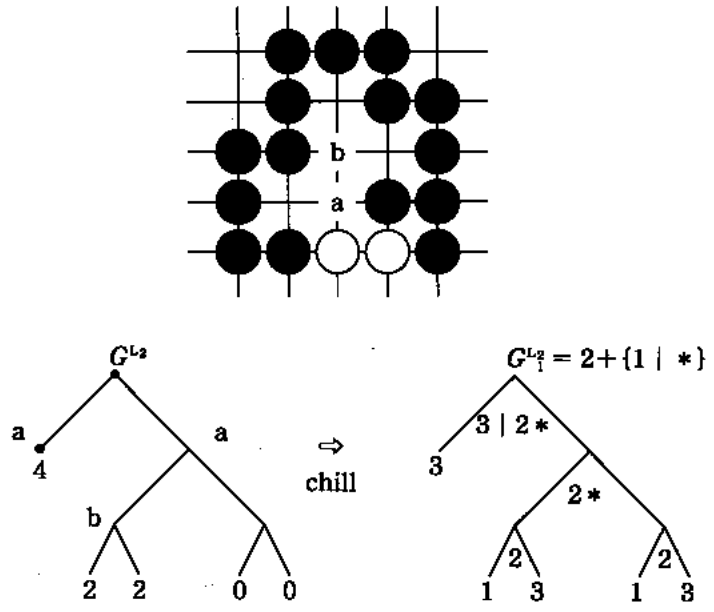


Figure 3.5

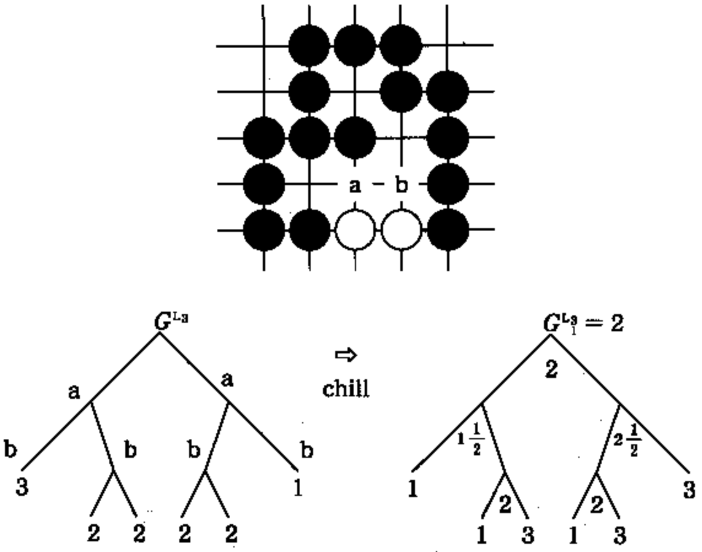


Figure 3.6

And Figures 3.7 through 3.9 show game tree analyses of chilled games:

$$G_1^{\mathrm{R}_1} = \tfrac{3}{4}, \quad \hat{G}_1^{\mathrm{R}_2} = (1+\uparrow)|1 = 1*,$$
$$\check{G}_1^{\mathrm{R}_2} = (1|0,\ \tfrac{1}{2})||1 = \tfrac{3}{4},$$
$$\hat{G}_1^{\mathrm{R}_3} = 3||2|1|||1 = 2|1, \quad \check{G}_1^{\mathrm{R}_3} = 3|0||1 = 0.$$

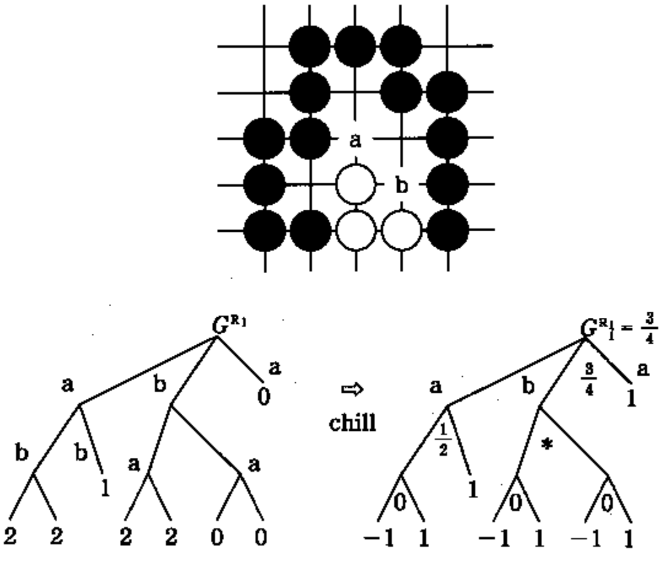


Figure 3.7

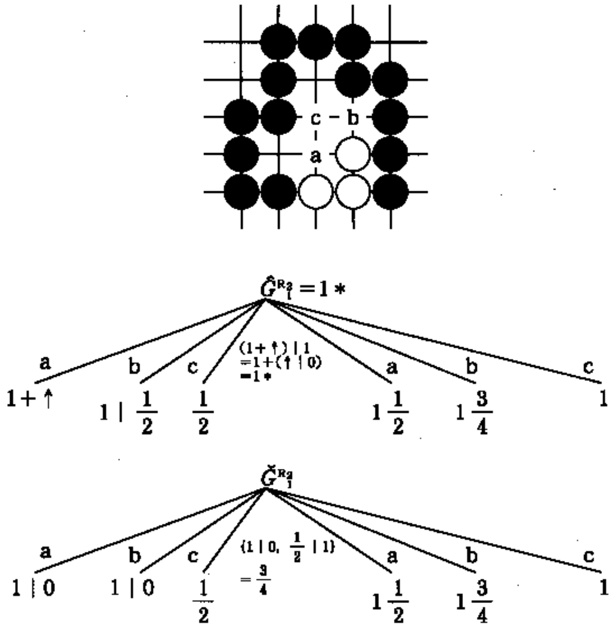


Figure 3.8

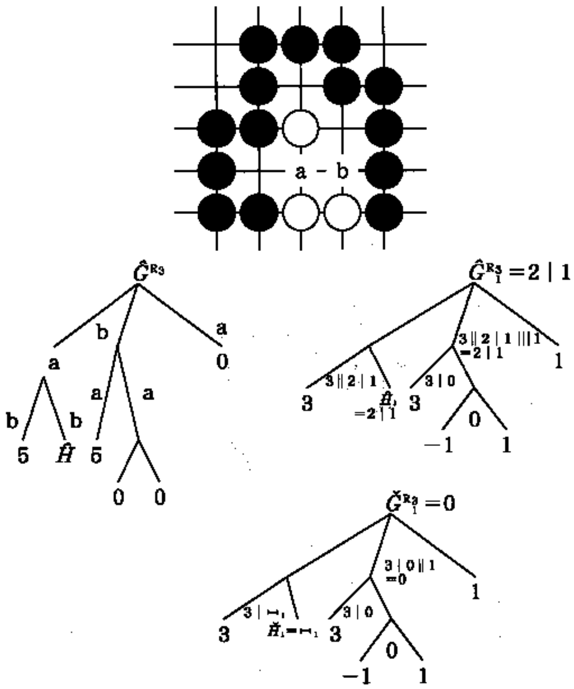


Figure 3.9

Finally, we get the analysis of the chilled game G itself as shown in Figure 3.10:

$$\hat{G}_1 = \{(2+\Uparrow *)|1\tfrac{3}{4}\}, \quad \check{G}_1 = \{(2+\!+_1)|1\} = 2|1.$$

Thus, Black should play a in Figure 3.1 regardless of who the ko-master is, but White should play a when Black is the ko-master and play c when White is the ko-master.

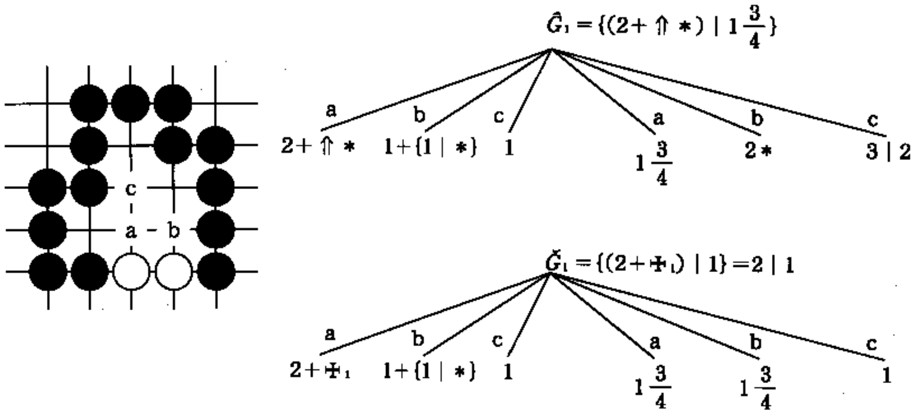


Figure 3.10

4. The Rogue Positions

Elwyn Berlekamp and David Wolfe have found an interesting Go position and named it the rogue position. This is now called Wolfe's Rogue Position and is shown in Figure 4.1. It is a width-one-entrance 7-room position. We then found a quite interesting width-two-entrance 7-room position and named it Takizawa's Rogue Position. This position is shown in Figure 4.2. There are two hidden kos in Takizawa's Rogue Position.

Black and White's options are shown in Figures 4.3 and 4.4. All but one, Black's option G^{L_3}, contain at least one ko in the game tree. The chilled game

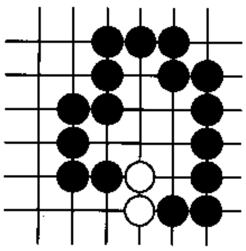


Figure 4.1

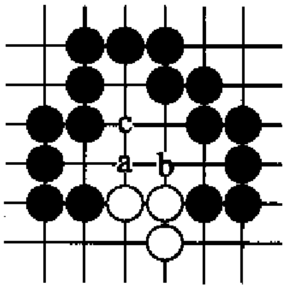


Figure 4.2

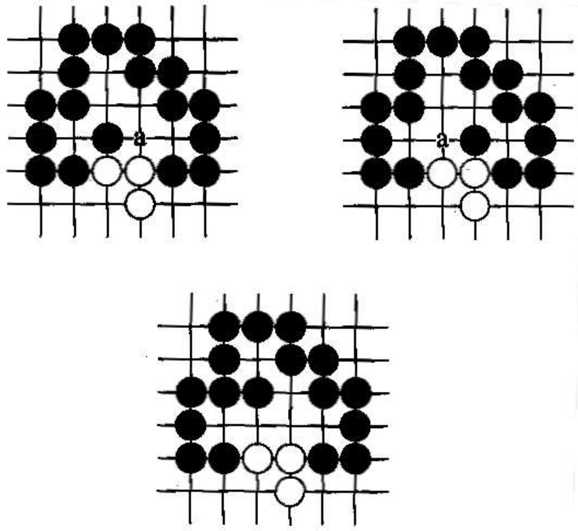


Figure 4.3

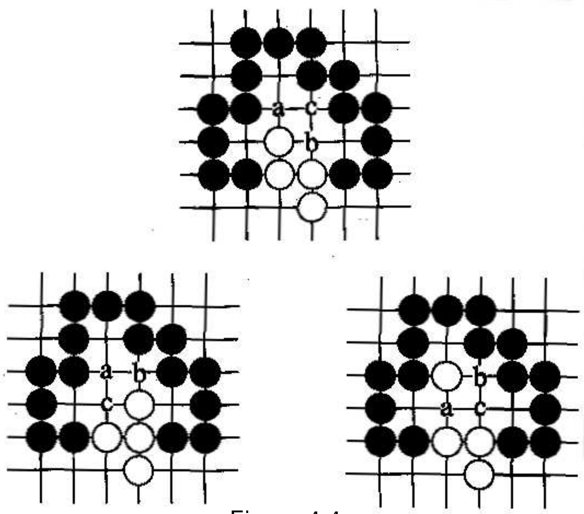


Figure 4.4

analyses are shown in Figures 4.5 through 4.10:

$$\begin{aligned}
\hat{G}_1^{\mathrm{L}_1} &= 3 + \{1|\uparrow\},\\
\check{G}_1^{\mathrm{L}_1} &= 2 + \{2||1|0\},\\
\hat{G}_1^{\mathrm{L}_2} &= 4||3|2\tfrac{1}{2},\\
\check{G}_1^{\mathrm{L}_2} &= 4||3|2\tfrac{1}{4},\\
G_1^{\mathrm{L}_3} &= 2 + \tfrac{1}{2}*,\\
\hat{G}_1^{\mathrm{R}_1} &= \check{G}_1^{\mathrm{R}_1} = 1\downarrow,\\
\hat{G}_1^{\mathrm{R}_2} &= 1*,\\
\check{G}_1^{\mathrm{R}_2} &= \{(\tfrac{1}{2}, 1|0)||1\},\\
\hat{G}_1^{\mathrm{R}_3} &= 4||2|1|||1 = 2|1,\\
\check{G}_1^{\mathrm{R}_3} &= 4|\tfrac{1}{4}||1 = \tfrac{1}{2}.
\end{aligned}$$

Finally, we get the analysis of chilled game G itself as shown in Figures 4.11 and 4.12:

$$\hat{G}_1 = 2 + \{1|\uparrow||(\downarrow,*)\} \quad \text{and} \quad \check{G}_1 = \{3||2|1\tfrac{1}{4}|||1\tfrac{1}{2}\} = 2|1\tfrac{1}{2}.$$

Thus, Black should play a in Figure 4.2 when Black is the ko-master but play b when White is the ko-master. White should play a or b when Black is the ko-master but play c when White is the ko-master. These are quite interesting results.

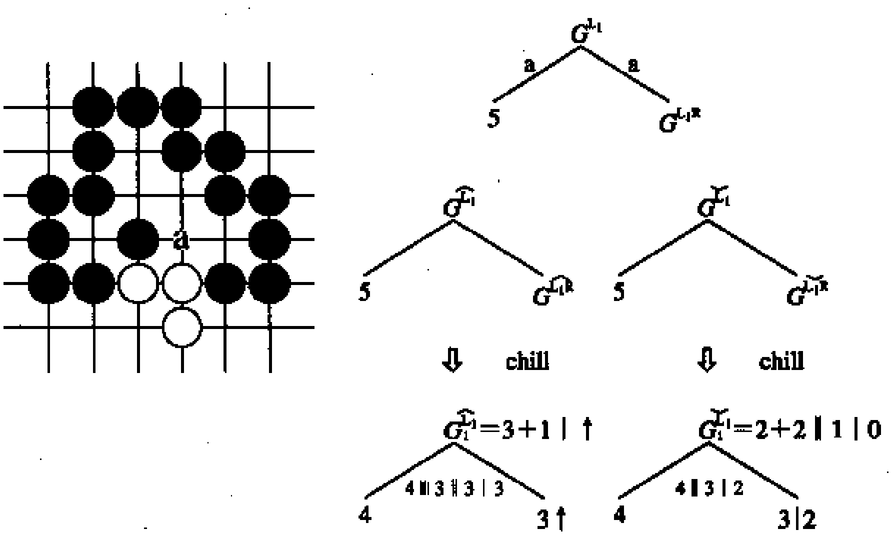


Figure 4.5

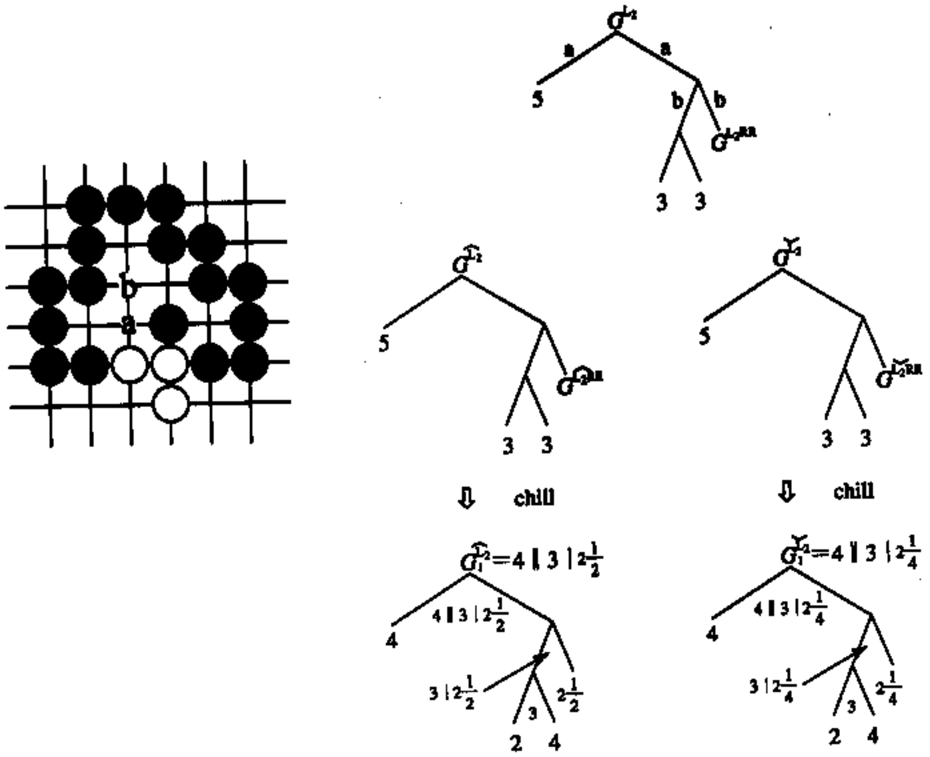


Figure 4.6

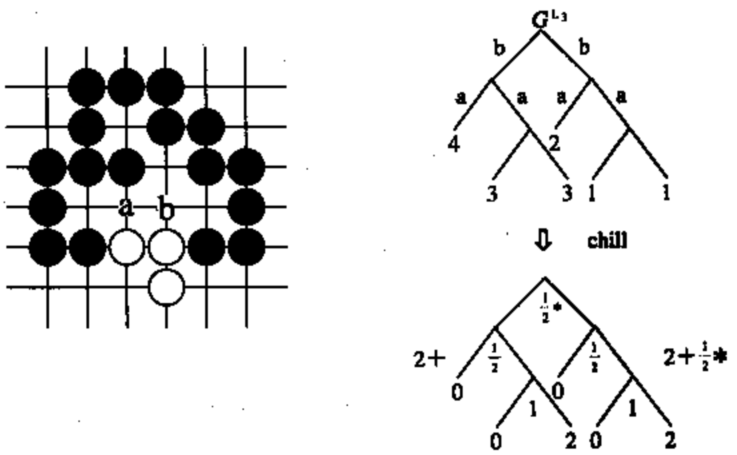
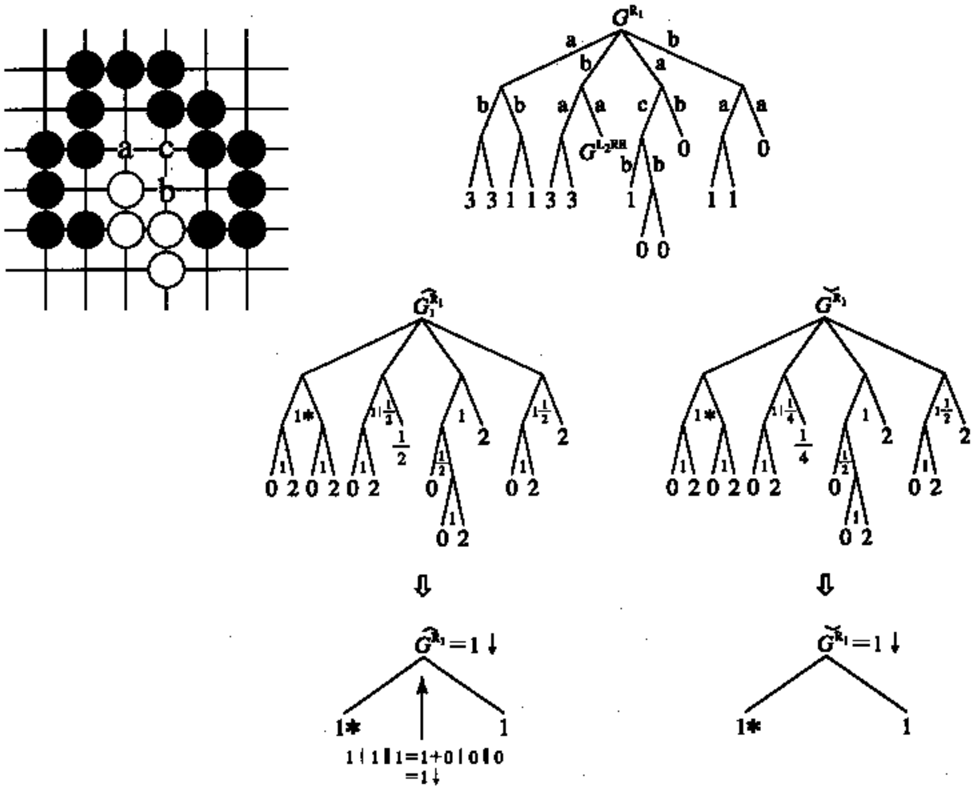


Figure 4.7



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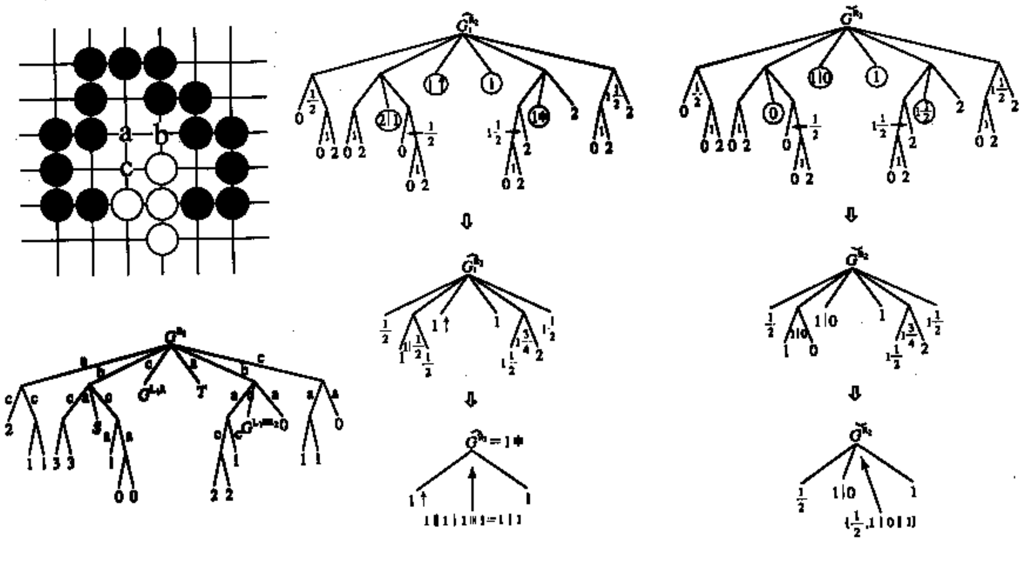

Figure 4.9

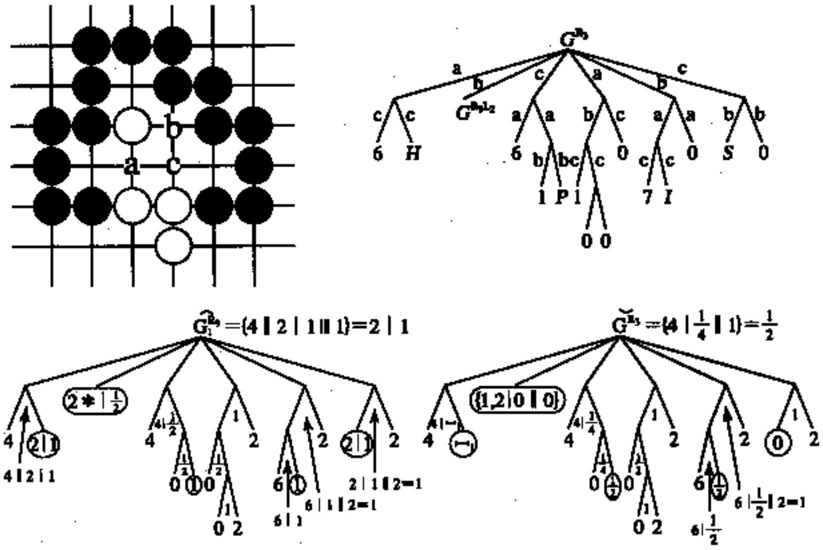

Figure 4.10

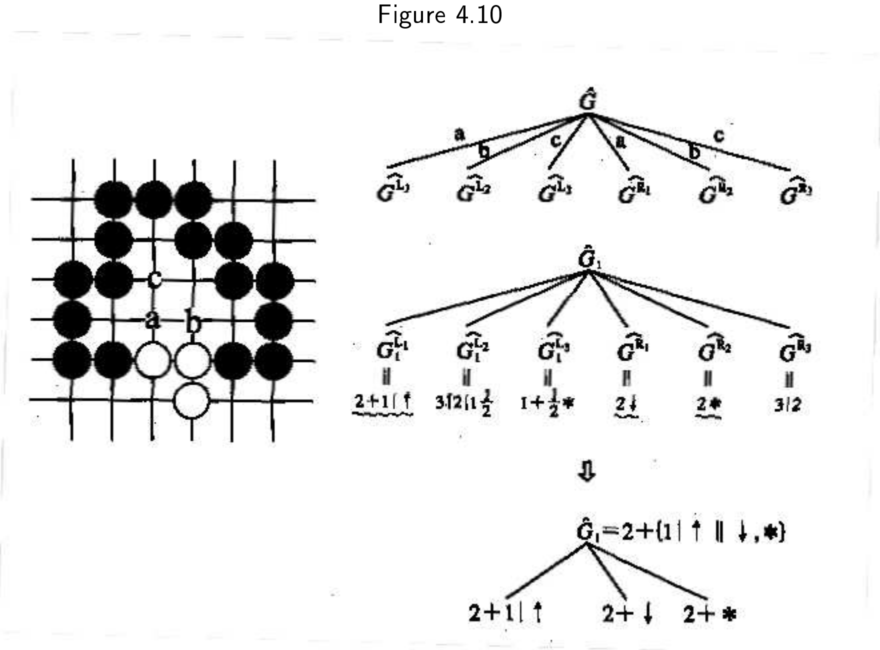

Figure 4.11

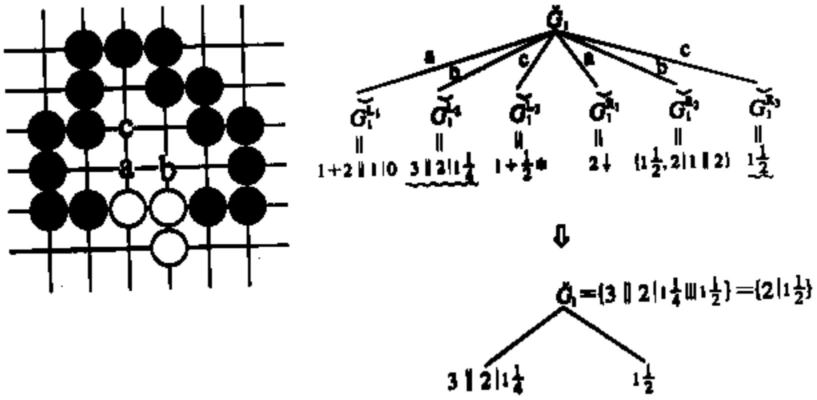

Figure 4.12

Acknowledgements and Conclusion

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