SOME CONDITIONS ON DOUGLAS ALGEBRAS THAT IMPLY THE INVARIANCE OF THE MINIMAL ENVELOPE MAP

CARROLL GUILLORY

Abstract. We give several conditions on certain families of Douglas algebras that imply that the minimal envelope of the given algebra is the algebra itself. We also prove that the minimal envelope of the intersection of two Douglas algebras is the intersection of their minimal envelope.

1. Introduction

Let D denote the open unit disk in the complex plane, and T the unit circle. By L^∞ we mean the space of essentially bounded measurable functions on T with respect to the normalized Lebesgue measure. We denote by H^∞ the space of all bounded analytic functions in D. Via identification with boundary functions, H^∞ can be considered as a uniformly closed subalgebra of L^∞. Any uniformly closed subalgebra B strictly between H^∞ and L^∞ is called a Douglas algebra. We denote by $M(B)$ the maximal ideal space of a Douglas algebra B. If C is the set of all continuous functions on T, we set $H^\infty + C = \{h + g : h \in H^\infty,\ g \in C\}$. Then $H^\infty + C$ becomes the smallest Douglas algebra containing H^∞ properly.

The function

$$q(z) = \prod_{n=1}^{\infty} \frac{|z_n|}{z_n} \frac{z - z_n}{1 - \bar{z}_n z}$$

is called a Blaschke product if $\sum_{n=1}^{\infty}(1 - |z_n|)$ converges. The set $\{z_n\}_n$ is called the zero set of q in D. Here $|z_n|/z_n = 1$ is understood whenever $z_n = 0$. We call q an interpolating Blaschke product if

$$\inf_n \prod_{m: m \neq n} \left| \frac{z_m - z_n}{1 - \bar{z}_n z_m} \right| > 0.$$

All work on this paper was done while the author was at the Mathematical Science Research Institute. The author thanks the Institute for its support during this period. Research at MSRI is supported in part by NSF grant DMS-9022140.

An interpolating Blaschke product q is called sparse (or thin) if

$$\lim_{n\to\infty}\prod_{m:m\neq n}\left|\frac{z_m-z_n}{1-\bar{z}_n z_m}\right|=1.$$

The set $Z(q)=\{x\in M(H^\infty)\setminus D : q(x)=0\}$ is called the zero set of q in $M(H^\infty+C)$. Any function h in H^∞ with $|h|=1$ almost everywhere on T is called an inner function. Since $|q|=1$ for any Blaschke product, Blaschke products are inner functions. Let $QC=(H^\infty+C)\cap\overline{(H^\infty+C)}$ and for $x\in M(H^\infty+C)$, set

$$Q_x=\{y\in M(L^\infty): f(x)=f(y) \text{ for all } f\in QC\}.$$

Then Q_x is called the QC-level set for x. For $x\in M(H^\infty+C)$, we denote by μ_x the representing measure for x, and its support set by $\operatorname{supp}\mu_x$. By $H^\infty[\bar{q}]$ we mean the Douglas algebra generated by H^∞ and the complex conjugate of the function q. Since $M(L^\infty)$ is the Shilov boundary for every Douglas algebra, a closed set E contained in $M(L^\infty)$ is called a peak set for a Douglas algebra B if there is a function f in B with $f=1$ on E and $|f|<1$ on $M(L^\infty)\setminus E$. A closed set E is a weak peak set for B if E is the intersection of a family of peak sets. If the set E is a weak peak set for H^∞ and we define

$$H^\infty_E=\{f\in L^\infty : f|_E\in H^\infty|_E\},$$

then H^∞_E is a Douglas algebra. For a Douglas algebra B, we define B_E similarly.

For an interpolating Blaschke product q we put $N(\bar{q})$ the closure of

$$\bigcup\{\operatorname{supp}\mu_x : x\in M(H^\infty+C),\ |q(x)|<1\}.$$

Then $N(\bar{q})$ is a weak peak set for H^∞. By $N_0(\bar{q})$ we denote the closure of $\bigcup\{\operatorname{supp}\mu_x : x\in Z(q)\}$. In general $N_0(\bar{q})$ does not equal $N(\bar{q})$, but in this paper $N_0(\bar{q})=N(\bar{q})$. For $x\in M(H^\infty)$, we let $E_x=\{y\in M(H^\infty):\operatorname{supp}\mu_y=\operatorname{supp}\mu_x\}$ and call E_x the level set of x. Since the sets $\operatorname{supp}\mu_x$ and $N(\bar{q})$ are weak peak sets for H^∞, both $H^\infty_{\operatorname{supp}\mu_x}$ and $H^\infty_{N(\bar{q})}$ are Douglas algebras. For the interpolating Blaschke product q, set

$$A=\bigcap_{x\in M(H^\infty+C)}\{H^\infty_{\operatorname{supp}\mu_x} : |q(x)|<1\}$$

and

$$A_0=\bigcap\{H^\infty_{\operatorname{supp}\mu_x} : x\in Z(q)\}.$$

Our assumptions on q throughout this paper imply that $H^\infty_{N(\bar{q})}=A=A_0$ (see [10]). For x and y in $M(H^\infty)$, the pseudohyperbolic distance ρ is defined by

$$\rho(x,y)=\operatorname{supp}\{|h(y)| : \|h\|_\infty\le 1,\ h\in H^\infty,\ h(x)=0\}.$$

For x and y in D, we have

$$\rho(x,y) = \left|\frac{x-y}{1-\bar{y}x}\right|.$$

For $x \in M(H^\infty)$, we define the Gleason part P_x of x by

$$P_x = \{y \in M(H^\infty) : \rho(x,y) < 1\}.$$

If $P_x \neq \{x\}$, then x is said to be a nontrivial point. For the definition of those interpolating Blaschke products that are of type G and of finite type G, see [8].

Let B be a Douglas algebra. The Bourgain algebra B_b of B relative to L^∞ is the set of f in L^∞ such that $\|ff_n + B\|_\infty \to 0$ for every sequence $\{f_n\}$ in B with $f_n \to 0$ weakly in B. An algebra B is called a minimal superalgebra of an algebra A if $A \subset B$ and $\operatorname{supp}\mu_x = \operatorname{supp}\mu_y$ for all $x, y \in M(A) \setminus M(B)$. The minimal envelope B_m of a Douglas algebra B is defined to be the smallest Douglas algebra that contains all the minimal superalgebras of B. The mapping that assigns to B the Douglas algebra B_m is called the minimal envelope map.

2. Conditions for $B_m = B$

We begin with the following theorem. The case for the Bourgain algebra B_b has been proven in [15]. The proof used here is quite different from theirs, and can be used to show that this result also holds for the Bourgain algebras.

Theorem 1. *Let A and B be Douglas algebras. Then $(A \cap B)_m = A_m \cap B_m$.*

Proof. Since $A \cap B$ is contained in both A and B by Proposition 6 of [11], $(A \cap B)_m$ is contained in both A_m and B_m. Hence $(A \cap B)_m \subset A_m \cap B_m$.

To show that $A_m \cap B_m \subset (A \cap B)_m$, let ψ be an interpolating Blaschke product such that $\bar{\psi} \in A_m \cap B_m$. We show that $\bar{\psi} \in (A \cap B)_m$. By Theorem D of [11] we can assume that there is an $x \in M(A)$ and a $y \in M(B)$ such that $\{\lambda \in M(A) : |\psi(\gamma)| < 1\} = E_x$ and $\{\mu \in M(B) : |\psi(\mu)| < 1\} = E_y$. This implies that $M(A) = M(A[\bar{\psi}]) \cup E_x$ and $M(B) = M(B[\bar{\psi}]) \cup E_y$. Thus

$$\begin{aligned} M(A \cap B) &= M(A) \cup M(B) = M(A[\bar{\psi}]) \cup M(B[\bar{\psi}]) \cup E_x \cup E_y \\ &= M(A[\bar{\psi}] \cap B[\bar{\psi}]) \cup E_x \cup E_y = M((A \cap B)[\bar{\psi}]) \cup E_x \cup E_y. \end{aligned}$$

Hence $\{\omega \in M(A \cap B) : |\psi(\omega)| < 1\} = E_x \cup E_y$. By Theorem 3 of [11] we have $\bar{\psi} \in (A \cap B)_m$. Our theorem follows. □

The following corollaries are immediate consequences of the theorem.

Corollary 1. *Let A and B be Douglas algebras with $A = A_m$ and $B = B_m$. Then $A \cap B = (A \cap B)_m$.*

Corollary 2. *Let B_0 be a Douglas algebra with $(B_0)_m = B_0$ and B be a Douglas algebra such that there is an interpolating Blaschke product q with $\{\lambda \in M(B) : |q(\lambda)| = 1\} \subset M(B_0)$. If A has the property that $A_m = A$, then $B[\bar{q}] \cap A = (B[\bar{q}] \cap A)_m$. In particular, $H^\infty[\bar{\varphi}] \cap A = (H^\infty[\bar{q}] \cap A)_m$ for every interpolating Blaschke product φ.*

Proof. The hypothesis $\{\lambda \in M(B) : |q(\lambda)| = 1\} \subset M(B_0)$ implies that $B_0[\bar{q}] = B[\bar{q}]$, hence $[B[\bar{q}]]_m = B[\bar{q}]$. So $B[\bar{q}] \cap A = (B[\bar{q}] \cap A)_m$ follows from our theorem. By Theorem 1 of [2] we have $H^\infty_m = H^\infty + C$, so $\{\lambda \in M(H^\infty) : |\psi(\lambda)| = 1\} \subset M(H^\infty + C)$ for every interpolating Blaschke product. The second part of the corollary follows. $\square$

Theorem 2. *Let B_0 be a Douglas algebra such that $(B_0)_m = B_0$. Let B be any other Douglas algebra such that there is an interpolating Blaschke product φ with $\{\lambda \in M(B) : |\varphi(\lambda)| = 1\} \subset M(B_0)$. For an interpolating Blaschke product q set $B_x = B[\bar{\varphi}] \cap H^\infty_{\operatorname{supp}\mu_x}$ for each $x \in Z(q)$. Put $B_e = \bigcap\{B_x : x \in Z(q)\}$.*

(i) *If q is of finite type G, then $(B_e)_b = B_e$.*
(ii) *If q is the product of a finite number of sparse interpolating Blaschke products, then $(B_e)_m = B_e$.*

Proof. We show that $B_e = B[\bar{\varphi}] \cap H^\infty_{N(\bar{q})}$. By, an unpublished result of D. Sarason,

$$\begin{aligned} M(B_e) &= M\Big(\bigcap_{x\in Z(q)} B_x\Big) = \overline{\bigcup_{x\in Z(q)} M(B_x)} \\ &= \overline{\bigcup_{x\in Z(q)} M(B[\bar{\varphi}] \cap H^\infty_{\operatorname{supp}\mu_x})} = \overline{\bigcup_{x\in Z(q)} (M(B[\bar{\varphi}]) \cup M(H^\infty_{\operatorname{supp}\mu_x}))}\,, \\ M(B_e) &= M(B[\bar{\varphi}]) \cup \overline{\bigcup_{x\in Z(q)} M(H^\infty_{\operatorname{supp}\mu_x})}\,. \end{aligned}$$

Now, if q is of finite type g, Proposition 1 of [9] and Theorem 3.2(i) of [8] imply $\bigcap_{x\in Z(q)} H^\infty_{\operatorname{supp}\mu_x} = H^\infty_{N(\bar{q})}$. Thus

$$\overline{\bigcup_{x\in Z(q)} M(H^\infty_{\operatorname{supp}\mu_x})} = M(H^\infty_{N(\bar{q})})$$

and we get

$$\begin{aligned} M(B_e) &= M(B[\bar{\varphi}]) \cup M(H^\infty_{N(\bar{q})}) \\ &= M(B[\bar{\varphi}] \cap H^\infty_{N(\bar{q})}). \end{aligned}$$

So by the Chang–Marshall Theorem [1, 13] we have $B_e = B[\bar{\varphi}] \cap H^\infty_{\operatorname{supp}\mu_x}$.

The condition $\{\lambda \in M(B) : |\varphi(\lambda)| = 1\} \subset M(B_0)$ implies that $B[\bar{\varphi}] = B_0[\bar{\varphi}]$. Hence $[B[\bar{\varphi}]]_m = [B_0[\bar{\varphi}]]_m = [B_0]_m[\bar{\varphi}] = B_0[\bar{\varphi}] = B[\bar{\varphi}]$, where the middle equality follows from Theorem 4 of [11]. By theorem 1 of [7] (and its proof) we have $(H^\infty_{N(\bar{q})})_b = H^\infty_{N(\bar{q})}$ if q is of finite type G. Thus

$$(B_e)_b = (B[\bar{\varphi}] \cap H^\infty_{N(\bar{q})})_b = B[\bar{\varphi}] \cap H^\infty_{N(\bar{q})} = B_e.$$

This proves (i).

Now if q is the product of a finite number of sparse Blaschke products, then again by Theorem 1 of [7] we have $(H^\infty_{N(\bar{q})})_m = H^\infty_{N(\bar{q})}$, and so $(B_e)_m = B_e$. This proves (ii). □

Corollary 3. *Suppose the hypothesis of Theorem 2 holds if $\varphi = q$. Then $B_e = B$, and:*

(i) $B = (B)_b$ *if q is of finite type G;*
(ii) $B = B_m$ *is the product of a finite number of sparse Blaschke products.*

Theorem 3. *Let B be a Douglas algebra and let E be a peak set for B. Then $(B_E)_m = B_E$.*

Proof. Since E is a peak set for B, there is a function $f \in B$ such that $f = 1$ on E and $|f| < 1$ on $M(L^\infty) \setminus E$. By [3, p. 39] we have

$$M(B_E) = \{x \in M(B) : \operatorname{supp} \mu_x \subseteq E\} \cup M(L^\infty) = \{x \in M(B) : f(x) = 1\} \cup M(L^\infty).$$

Let I be any interpolating Blaschke product such that $Z(I) \cap M(B_E) \neq \varnothing$. We will show that there is an uncountable set $\Gamma \subset Z(I) \cap M(B_E)$ such that $E_\alpha \neq E_\beta$ for distinct $\alpha, \beta \in \Gamma$. Let $\{z_n\}$ be the zero sequence of I in D and $y \in Z(I) \cap M(B_E)$. Since $y \in Z(I)$, there is a subsequence $\{z_{n_k}\}$ of $\{z_n\}$ such that $z_{n_k} \to y$. Since $y \in M(B_E)$, we have $f(z_{n_k}) \to 1$. Let I_1 be the factor of I such that $\{z_{n_k}\}$ is the zero sequence of I_1. Then

$$Z(I_1) = \overline{\{z_{n_k}\}} \setminus \{z_{n_k}\}$$

and since $Z(I_1)$ is equivalent to the Čech compactification of the integers, $Z(I_1)$ is an uncountable set. Since $f(z_{n_k}) \to 1$, we have that $f = 1$ on $Z(I_1)$. Thus $Z(I_1) \subset Z(I) \cap M(B_E)$, and so $Z(I) \cap M(B_E)$ is an infinite set. To show that Γ exists, take a subsequence $\{z_{n_{k_0}}\}$ of $\{z_{n_k}\}$ such that $\{z_{n_{k_0}}\}$ is a sparse Blaschke sequence in D (see [4]). Let I_0 be the sparse Blaschke product whose zero sequence in D is $\{z_{n_{k_0}}\}$. Then, by Theorem 1 of [12], we have

$$N(I_0) = \bigcup_{x \in Z(I_0)} Q_x.$$

Since $Z(I_0)$ is uncountable, Lemma 4 of [12] says that $Q_x \neq Q_y$ for distinct $x, y \in Z(I_0)$. This implies that $\operatorname{supp}\mu_x \cap \operatorname{supp}\mu_y = \varnothing$ and $E_x \neq E_y$. Set $\Gamma = Z(I_0)$. Then

$$\bigcup_{\alpha\in\Gamma} E_\alpha \subseteq \{\lambda \in M(B_E) : |I(\lambda)| < 1\}.$$

By Theorem 3 of [11], $\bar{I} \notin (B_E)_m$. This shows that $(B_E)_m = B_E$. □

Theorem 4. *Let A be a Douglas algebra and let $\{q_n\}$ be a sequence of interpolating Blaschke products such that $\overline{q_n} \notin A$ for every n. Let $B = A[\overline{q_n}; n = 1, 2, 3 \dots]$. Then $B_m = B$.*

Proof. Since $M(B) = \{\lambda \in M(A) : |q_{n(\lambda)}| = 1,\, n = 1, 2, \dots\}$, we can define the function F on $M(H^\infty)$ as

$$F(x) = \sum_{n=1}^{\infty} \frac{1}{2^n} |q_n(x)|.$$

Then $F = 1$ on $M(B)$ and $|F| < 1$ on $M(H^\infty) \setminus M(B)$. Let C be any interpolating Blaschke product such that $Z(C) \cap M(B) \neq \varnothing$. Let $\{z_n\}$ be the zero sequence of C in D. As in the proof of Theorem 3 we can find a sparse subsequence $\{z_{n_k}\}$ of $\{z_n\}$ such that $F = 1$ on $\overline{\{z_{n_k}\}} \setminus \{z_{n_k}\}$. If we set C_1 to be the factor of C whose zero sequence in D is $\{z_{n_k}\}$, then as in the proof of Theorem 3 we can show that $E_\alpha \neq E_\beta$ for distinct $\alpha, \beta \in Z(C_1)$, and

$$\bigcup_{\alpha\in Z(C_1)} E_\alpha \subset \{\lambda \in M(B) : |C\lambda| < 1\}.$$

So, by Theorem 3 of [11], we have $\bar{C} \notin B_m$. This implies that $B_m = B$. □

Theorem 5. *Let $\{q_n\}$ be a sequence of interpolating Blaschke products not invertible in a Douglas algebra A. Suppose that for each n there is a Douglas algebra A_n with $(A_n)_m = A_n$ and*

$$\{\lambda \in M(A) : |q_n(\lambda)| = 1\} \subset M(A_n).$$

Let $B = \bigcap_{n=1}^{\infty} A[\bar{q}_n]$. Then $B_m = B$.

Proof. Since $B \subseteq A[\bar{q}_n]$ for $n = 1, 2 \dots$, by Proposition 6 of [11] we have $B_m \subseteq (A[\bar{q}_n])_m$. Hence

$$B_m \subseteq \bigcap_{n=1}^{\infty} (A[\overline{q}_n])_m.$$

To show that $B_m = B$ it suffices to show that $(A[\bar{q}_n])_m = A[\bar{q}_n]$ for all n. Since $\{\lambda \in M(A) : |q_n(\lambda)| = 1\} \subset M(A_n)$, we have $A[\bar{q}_n] = A_n[\bar{q}_n]$. So using Theorem 4 of

[11] we have

$$(A[\bar{q}_n])_m = (A_n[\bar{q}_n])_m = (A_n)_m[\bar{q}_n] = A_n[\bar{q}_n] = A[\bar{q}_n].$$

Thus $B_m \subseteq \bigcap_{n=1}^{\infty}(A[\bar{q}_n])_m = \bigcap_{n=1}^{\infty} A[\bar{q}_n] = B$. □

Corollary 4. *If $B = \bigcap_{n=1}^{\infty} H^\infty[\bar{q}_n]$, then $B_m = B$.*

Theorem 6. *Let B be a Douglas algebra. Then $B \subset B_m$ if and only if there exists a point $x \in M(B)$ whose level set E_x is an open subset in $M(B)$.*

Proof. Let $B \subset B_m$. By Theorem D of [11] there exists an interpolating Blaschke product φ such that $\{\lambda \in M(B) : |\varphi(\lambda)| < 1\} = E_x$ for some $x \in M(B)$. By Lemma 9(ii) of [11], E_x is an open subset of $M(B)$.

Suppose that $B = B_m$. Then for every $y \in M(B)$ and every interpolating Blaschke product q with $q(y) = 0$ the set $Z(q) \cap M(B)$ contains an infinite subset Γ such that $E_\alpha \neq E_\beta$ for all distinct $\alpha, \beta \in \Gamma$, and

$$\bigcup_{\alpha \in \Gamma} E_\alpha = \{\lambda \in M(B) : |q(\lambda)| < 1\}.$$

We'll show that there is a subset $\{x_\sigma\}_{\sigma \in A}$ of Γ such that $y \in \overline{\{x_\sigma\}}_{\sigma \in A} \setminus \{y\}$, that is,

$$E_y \subseteq \overline{\{E_\alpha\}}_{\alpha \in \{\Gamma \setminus \{y\}\}}.$$

Suppose to the contrary that $E_y \not\subset \overline{\{E_\alpha\}}_{\alpha \in \{\Gamma \setminus \{y\}\}}$. Then $\overline{\{E_\alpha\}}_{\alpha \in \{\Gamma \setminus \{y\}\}}$ and $\{y\}$ are two closed disjoint subsets of $M(H^\infty)$. Choose neighborhoods V of $\overline{\{E_\alpha\}}_{\alpha \in \{\Gamma \setminus \{y\}\}}$ and U of $\{y\}$ such that $\bar{U} \cap \bar{V} = \varnothing$. Let q_1 be the the factor of q with zeros in $U \cap D$. Then we see that $Z(q_1) \cap M(B) \subset E_y$ and $\{\lambda \in M(B) : |q_1(\lambda)| < 1\} = E_y$. Thus $\bar{q}_1 \in B_m \setminus B$, which is a contradiction. Hence

$$E_y \subset \overline{\{E_\alpha\}}_{\alpha \in \{\Gamma \setminus \{y\}\}}.$$

Thus there is a subset $\{x_\sigma\}_{\sigma \in A} \subset \Gamma$ such that $x_\sigma \to y$, and $x_\sigma \notin E_y$ for all $\sigma \in A$. This implies that $M(B) \setminus E_y$ is not a closed subset of $M(H^\infty)$. Thus E_y is not an open subset of $M(B)$. This proves our theorem. □

I have been unable to answer the following two questions, which I close with.

Question 1. *Find an $x \in M(H^\infty + C) \setminus M(L^\infty)$ such that $H^\infty_{\operatorname{supp}\mu_x} = (H^\infty_{\operatorname{supp}\mu_x})_m$ (if such an x exists).*

Question 2. *Find a Douglas algebra B such that $B \subsetneq B_b \subsetneq B_m$ (if such a Douglas algebra exists).*

An immediate consequence of Theorem 6 above and Theorem 7 of [14], related to Question 2, is the following:

Theorem 7. *Let B be a Douglas algebra. Then $B = B_b \subsetneq B_m$ if and only if for all $x \in M(B)$ the set P_x is not open in $M(B)$ but for some $x \in M(B)$ the set E_x is an open subset in $M(B)$.*

References

[1] S.-Y. Chang, "A Characterization of Douglas subalgebras," *Acta. Math.* 137 (1976), 81–89.

[2] J. Cima, S. Janson, and K. Yale, "Completely continuous Hankel operators on H^∞ and Bourgain algebras," *Proc. Amer. Math. Soc.* 105 (1989), 121–125.

[3] T. W. Gamelin, *Uniform Algebras*, New York, Chelsea Publishing Company, 1984.

[4] P. Gorkin, "Decomposition of the maximal ideal space of L^∞," Thesis, Michigan State Univ., East Lansing, MI, 1982.

[5] P. Gorkin, K. Izuchi, and R. Mortini, "Bourgain algebras of Douglas algebras," *Canad. J. Math.* 44 (1992), 797–804.

[6] P. Gorkin, H.-M. Lingenberg, and R. Mortini, "Homeomorphic disks in the spectrum of H^∞," *Indiana Univ. Math. J.* 39 (1990), 961–983.

[7] C. J. Guillory, "Bourgain algebras, minimal envelopes, and minimal support sets," MSRI Preprints, 1995.

[8] C. J. Guillory and K. Izuchi, "Interpolating Blaschke products of type G," to appear in *J. Complex Variables.*

[9] C. J. Guillory and K. Izuchi, "Maximal Douglas subalgebras and minimal support points," *Proc. Amer. Math. Soc.* 116 (1992), 477–481.

[10] C. J. Guillory and K. Izuchi, "Interpolating Blaschke products and nonanalytic sets," *J. Complex Variables* 23 (1993), 163–175.

[11] C. J. Guillory and K. Izuchi, "Minimal envelopes of Douglas algebras and Bourgain algebras," *Houston J. Math.* 19 (1993), 201–222.

[12] K. Izuchi, "QC-level sets and quotients of Douglas algebras," *J. Funct. Anal.* 65 (1986), 293–308.

[13] D. Marshall, "Subalgebras of L^∞ containing H^∞," *Acta. Math.* 137 (1976), 91–98.

[14] R. Mortini and R. Younis, "Douglas algebras which are invariant under the Bourgain map," *Arch. Math.* 59 (1992), 371–378.

[15] R. Younis and D. Zheng, "A distance formula and Bourgain algebras," preprint.

Department of Mathematics, University of Southwestern Louisiana, Lafayette, LA 70504

Current address: Mathematical Science Research Institute, 100 Centennial Drive, Berkeley, CA 94720