FINITELY SMOOTH REINHARDT DOMAINS WITH NON-COMPACT AUTOMORPHISM GROUP

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Abstract. We give a complete description of bounded Reinhardt domains of finite boundary smoothness that have non-compact automorphism group. As part of this program, we show that the classification of domains with non-compact automorphism group and having only finite boundary smoothness is considerably more complicated than the classification of such domains that have infinitely smooth boundary.

Let $D \subset \mathbb{C}^n$ be a bounded domain, and suppose that the group $\mathrm{Aut}(D)$ of holomorphic automorphisms of D is non-compact in the topology of uniform convergence on compact subsets of D. This means that there exist points $q \in \partial D$, $p \in D$ and a sequence $\{f_j\} \subset \mathrm{Aut}(D)$ such that $f_j(p) \to q$ as $j \to \infty$.

We also assume that D is a Reinhardt domain, i.e. that the standard action of the n-dimensional torus $\mathbb{T}^n$ on $\mathbb{C}^n$,

$$z_j \mapsto e^{i\phi_j} z_j, \qquad \phi_j \in \mathbb{R}, \quad j = 1, \dots, n,$$

leaves D invariant.

In [FIK1] we gave a complete classification of bounded Reinhardt domains with non-compact automorphism group and C^∞-smooth boundary. For the sake of completeness we quote the main result of [FIK1] below:

Theorem 1. *If D is a bounded Reinhardt domain in $\mathbb{C}^n$ with C^∞-smooth boundary, and if $\mathrm{Aut}(D)$ is not compact then, up to dilations and permutations of coordinates, D is a domain of the form*

$$\left\{ |z^1|^2 + \sum_{j=2}^{p} |z^j|^{2m_j} + P(|z^2|, \dots, |z^p|) < 1 \right\},$$

where P is a polynomial:

$$P(|z^2|, \dots, |z^p|) = \sum_{l_2, \dots, l_p} a_{l_2, \dots, l_p} |z^2|^{2l_2} \dots |z^p|^{2l^p}, \tag{1}$$

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$a_{l_2,\dots,l_p}$ are real parameters, $m_j \in \mathbb{N}$, with the sum taken over all $(p-1)$-tuples $(l_2,\dots,l_p)$, $l_j \in \mathbb{Z}$, $l_j \geq 0$, where at least two entries are non-zero, such that $\sum_{j=2}^{p} \frac{l_j}{m_j} = 1$, and the complex variables $z_1,\dots,z_n$ are divided into p non-empty groups $z^1,\dots,z^p$. In addition, the polynomial

$$\tilde{P}(|z^2|,\dots,|z^p|) = \sum_{j=2}^{p} |z^j|^{2m_j} + P(|z^2|,\dots,|z^p|)$$

is non-negative in $\mathbb{C}^{n-n_1}$ (n_1 is the number of variables in the group z^1), and the domain

$$\left\{(z^2,\dots,z^p) \in \mathbb{C}^{n-n_1} : \tilde{P}(|z^2|,\dots,|z^p|) < 1\right\}$$

is bounded.

In this paper we generalize Theorem 1 to the case when the boundary of the domain is only C^k-smooth, $k \geq 1$. To the best of our knowledge, this is the first attempt to obtain a general result for bounded domains with non-compact automorphism group and boundary of finite smoothness.

First of all, we note that, up to a certain point, the proof of Theorem 1 in [FIK1] is valid for domains with only C^1-smooth boundary. The C^∞-assumption was only used in Lemmas 1.6 and 1.8 of [FIK1]. Therefore, the proof of Theorem 1 in [FIK1] also gives the following proposition:

Proposition 2. *If D is a bounded Reinhardt domain in $\mathbb{C}^n$ with C^k-smooth boundary, $k \geq 1$, and if $\mathrm{Aut}(D)$ is not compact, then, by suitable dilations and permutations of coordinates, the domain D is equivalent to a Reinhardt domain G such that:*

(i) The set A of all points $(z^1, 0, \dots, 0)$, with $|z^1| = 1$, lies in ∂G.

(ii) In a neighbourhood of A, G is written in the form

$$\left\{|z^1|^2 + \phi(|z^2|,\dots,|z^p|) < 1\right\}, \tag{2}$$

where $\phi(x_2,\dots,x_p)$ is a non-negative C^k-smooth function in a neighbourhood of the origin in $\mathbb{R}^{p-1}$ such that $\phi(|z^2|,\dots,|z^p|)$ is also C^k-smooth in a neighbourhood of the origin in $\mathbb{C}^{n-n_1}$, and such that

$$\phi\left(t^{\frac{1}{\alpha_2}} x_2,\dots,t^{\frac{1}{\alpha_p}} x_p\right) = t\phi(x_2,\dots,x_p) \tag{3}$$

near the origin in $\mathbb{R}^{p-1}$ for $1 \leq t \leq 1+\epsilon$ and some $\epsilon > 0$. Here $\alpha_j > 0$, $j = 2,\dots,p$, and each α_j is either an even integer or, if it is not an even integer, then $\alpha_j > 2k$. In addition, the function ϕ satisfies

$$\phi(0,\dots,0,|z^j|,0,\dots,0) = |z^j|^{\alpha_j},$$

for $j = 2, \dots, p$.
(iii) G has the form

$$G = \left\{ (z^1, \dots, z^p) \in \mathbb{C}^n : |z^1| < 1, \left(\frac{z^2}{(1-|z^1|^2)^{\frac{1}{\alpha^2}}}, \dots, \frac{z^p}{(1-|z^1|^2)^{\frac{1}{\alpha^p}}} \right) \in \tilde{G} \right\}, \quad (4)$$

where $\tilde{G}$ is a bounded Reinhardt domain in $\mathbb{C}^{n-n_1}$.

We are now going to derive from Proposition 2 the following theorem, which is the main result of the present paper.

Theorem 3. *If D is a bounded Reinhardt domain in $\mathbb{C}^n$ with C^k-smooth boundary, $k \geq 1$, and if $\mathrm{Aut}(D)$ is not compact, then, up to dilations and permutations of coordinates, D is a domain of the form*

$$\left\{ |z^1|^2 + \psi(|z^2|, \dots, |z^p|) < 1 \right\}, \quad (5)$$

where $\psi(x_2, \dots, x_p)$ is a non-negative C^k-smooth function in $\mathbb{R}^{p-1}$ such that $\psi(|z^2|, \dots, |z^p|)$ is C^k-smooth in $\mathbb{C}^{n-n_1}$, and

$$\psi\left(t^{\frac{1}{\alpha_2}} x_2, \dots, t^{\frac{1}{\alpha_p}} x_p\right) = t\psi(x_2, \dots, x_p) \quad (6)$$

in $\mathbb{R}^{p-1}$ for all $t \geq 0$. Here $\alpha_j > 0$, $j = 2, \dots, p$, and each α_j is either an even integer or, if it is not an even integer, then $\alpha_j > 2k$. In addition, the function ψ satisfies

$$\psi(0, \dots, 0, |z^j|, 0, \dots, 0) = |z^j|^{\alpha_j}, \quad (7)$$

for $j = 2, \dots, p$, and the domain

$$\left\{ (z^2, \dots, z^p) \in \mathbb{C}^{n-n_1} : \psi(|z^2|, \dots, |z^p|) < 1 \right\} \quad (8)$$

is bounded.

Proof. First of all, using the weighted homogeneity property (3), we extend the function ϕ from a neighbourhood of the origin (see (2)) to a C^k-smooth function ψ on $\mathbb{R}^{p-1}$. Consider the surface

$$S_\delta = \{ |x_2|^{\alpha_2} + \dots + |x_p|^{\alpha_p} = \delta \},$$

and choose $\delta > 0$ such that S_δ lies in the neighbourhood of the origin in $\mathbb{R}^{p-1}$ where ϕ is defined and of class C^k and where (3) holds for $1 \leq t \leq 1 + \epsilon$. Further let

$$S_\delta^- = \{ |x_2|^{\alpha_2} + \dots + |x_p|^{\alpha_p} \leq \delta \},$$

and

$$S_\delta^+ = \{ |x_2|^{\alpha_2} + \dots + |x_p|^{\alpha_p} \geq \delta \}.$$

We now define the extension ψ for any $x=(x_2,\dots,x_p)\in\mathbb{R}^{p-1}$ as follows:

$$\psi(x)=\begin{cases}\phi(x), \text{ if } x\in S_\delta^-;\\ \frac{|x_2|^{\alpha_2}+\dots+|x_p|^{\alpha_p}}{\delta}\times\\ \times\phi\left(\frac{x_2\delta^{\frac{1}{\alpha_2}}}{(|x_2|^{\alpha_2}+\dots+|x_p|^{\alpha_p})^{\frac{1}{\alpha_2}}},\dots,\frac{x_p\delta^{\frac{1}{\alpha_p}}}{(|x_2|^{\alpha_2}+\dots+|x_p|^{\alpha_p})^{\frac{1}{\alpha_p}}}\right), \text{ if } x\in S_\delta^+.\end{cases}\tag{9}$$

Because of property (3), this definition implies that, for any $x\neq 0$

$$\begin{aligned}\psi(x)=&\frac{|x_2|^{\alpha_2}+\dots+|x_p|^{\alpha_p}}{\delta}\times\\ &\times\phi\left(\frac{x_2\delta^{\frac{1}{\alpha_2}}}{(|x_2|^{\alpha_2}+\dots+|x_p|^{\alpha_p})^{\frac{1}{\alpha_2}}},\dots,\frac{x_p\delta^{\frac{1}{\alpha_p}}}{(|x_2|^{\alpha_2}+\dots+|x_p|^{\alpha_p})^{\frac{1}{\alpha_p}}}\right),\end{aligned}\tag{10}$$

and $\psi(0)=0$. Further, since, for $j=2,\dots,p$, α_j is either a positive integer, or, if not, $\alpha_j>2k$, one has that $\psi\in C^k\left(\mathbb{R}^{p-1}\right)$ and $\psi(|z^2|,\dots,|z^p|)\in C^k\left(\mathbb{C}^{n-n_1}\right)$. Next, (10) implies that ψ has property (6) for all $x\in\mathbb{R}^{p-1}$ and $t\geq 0$, as well as property (7). It is also clear that $\psi\geq 0$.

We will now show that the domain G has the form (5), with ψ defined in (9). Let U be a neighbourhood of the set A (see (i) of Proposition 2) such that $G\cap U$ is given by (2). We can assume that $U=V\times W$, where V is a neighbourhood of the unit sphere in $\mathbb{C}^{n_1}$, and W is a neighbourhood of the origin in $\mathbb{C}^{n-n_1}$. Take $\sigma>0$ and consider $G_\sigma=G\cap\{|z^1|^2=1-\sigma\}$. Since $\tilde G$ is bounded, representation (4) implies that, if σ is sufficiently small, $G_\sigma\subset U$, and $\overline{G_\sigma}$ is a compact subset of U. It then follows from (2) that G_σ is given by

$$G_\sigma=\left\{(z^1,\dots,z^p)\in U\colon |z^1|^2=1-\sigma,\ \phi(|z^2|,\dots,|z^p|)<\sigma\right\},$$

and the set $\{(z^2,\dots,z^p)\in W\colon \phi(|z^2|,\dots,|z^p|)\leq\sigma\}$ is compact in W.

Further, since the extension ψ of ϕ has property (6), G_σ can be rewritten as

$$G_\sigma=\left\{(z^1,\dots,z^p)\in\mathbb{C}^n: |z^1|^2=1-\sigma,\ \psi(|z^2|,\dots,|z^p|)<\sigma\right\}.$$

On the other hand, (4) gives

$$G_\sigma=\left\{(z^1,\dots,z^p)\in\mathbb{C}^n: |z^1|^2=1-\sigma,\ \left(\frac{z^2}{\sigma^{\frac{1}{\alpha^2}}},\dots,\frac{z^p}{\sigma^{\frac{1}{\alpha^p}}}\right)\in\tilde G\right\},$$

which implies that

$$\tilde G=\left\{(z^2,\dots,z^p)\in\mathbb{C}^{n-n_1}: \psi(\sigma^{\frac{1}{\alpha_2}}|z^2|,\dots,\sigma^{\frac{1}{\alpha_p}}|z^p|)<\sigma\right\}.$$

It now follows from homogeneity property (6) for ψ that

$$\tilde G=\left\{(z^2,\dots,z^p)\in\mathbb{C}^{n-n_1}: \psi(|z^2|,\dots,|z^p|)<1\right\}.$$

Now (4) and (6) imply that G is in fact given by formula (5).

Finally, domain (8) is bounded since it coincides with $\tilde{G}$.

The theorem is proved. ■

For Reinhardt domains in $\mathbb{C}^2$, one has either $p = 1$ or $p = 2$. If $p = 1$, then domain (5) is the unit ball. If $p = 2$, then, because of (7), the function ψ from (5) has the form $\psi = |z_2|^\alpha$, $\alpha > 0$. This observation gives the following corollary.

Corollary 4. *If D is a bounded Reinhardt domain in $\mathbb{C}^2$ with C^k-smooth boundary, $k \geq 1$, and if* $\mathrm{Aut}(D)$ *is not compact, then, up to dilations and permutations of coordinates, D has the form*

$$\{|z_1|^2 + |z_2|^\alpha < 1\}, \tag{11}$$

where $\alpha > 0$ and either is an even integer or, if it is not an even integer, then $\alpha > 2k$.

Remark. Note that Corollary 4 is reminiscent of a result of Bedford/Pinchuk (see [BP1]): a pseudoconvex smoothly bounded domain in $\mathbb{C}^2$ with non-compact automorphism group and boundary of finite type in the sense of Kohn must be biholomorphic to a domain of the form (11) where α *is an even integer*. The results of Bedford/Pinchuk, and related conjectures, are discussed in more details at the end of this paper.

Theorem 3 reduces the classification problem for Reinhardt domains with non-compact automorphism group and C^k-smooth boundary to the problem of describing C^k-smooth functions ψ as in (5) that satisfy weighted homogeneity condition (6). For $p \geq 3$, one can construct examples of such functions in the following manner. Consider the following set of $(p-1)$-tuples $s = (s_2, \ldots, s_p)$

$$M = \Big\{ s = (s_2, \ldots, s_p) \in \mathbb{R}^{p-1} : s_j \geq 0;\ \text{each } s_j \text{ is either an even integer, or, if it is not an even integer, then } s_j > 2k;\ s \text{ has at least two non-zero entries; and } \sum_{j=2}^{p} \frac{s_j}{\alpha_j} = 1 \Big\}. \tag{12}$$

Let μ be an arbitrary finite measure on the set M. Then the function

$$\psi(|z^2|, \ldots, |z^p|) = \sum_{j=2}^{p} |z^j|^{\alpha_j} + \int_M |z^2|^{s_2} \ldots |z^p|^{s_p}\, d\mu \tag{13}$$

has all the properties as stated in Theorem 3 above, provided $\psi \geq 0$ and the corresponding domain (8) is bounded.

We now give an explicit non-trivial example of a function of the form (13).

Example 5. Consider the case of $\mathbb{C}^3$ and let $p = 3$, i.e. $z^j = z_j$, $j = 1, 2, 3$. Let $k = 2$, $\alpha_2 = \alpha_3 = 9$. Then it follows from (12) that

$$M = \{(s_2, s_3) \in \mathbb{R}^2 : s_2 = 9 - s_3,\ 4 \le s_3 \le 5\} \cup \{(2, 7)\} \cup \{(7, 2)\}.$$

We interpret M as a subset of $\mathbb{R}$ parametrized by s_3 and let $d\mu = ds$ be the usual Lebesgue measure on $\mathbb{R}$. Then the function defined by (13) becomes

$$\psi(|z_2|, |z_3|) = |z_2|^9 + |z_3|^9 + |z_2|^9 \int_4^5 \frac{|z_3|^s}{|z_2|^s}\, ds =$$
$$|z_2|^9 + |z_3|^9 + \frac{1}{\log|z_3|^2 - \log|z_2|^2}\left(|z_2|^4|z_3|^5 - |z_2|^5|z_3|^4\right). \quad (14)$$

The last term in function (14) and its first and second derivatives are defined to be equal to zero whenever $z_2 = 0$, or $z_3 = 0$, or $|z_2| = |z_3|$.

One can check directly that function (14) is indeed non-negative, C^2-smooth, has an appropriate homogeneity property (6) with $\alpha_2 = \alpha_3 = 9$, and the corresponding domain (8) is bounded. The Reinhardt domain $D \subset \mathbb{C}^3$ given by

$$D = \left\{|z_1|^2 + |z_2|^9 + |z_3|^9 + \frac{1}{\log|z_3|^2 - \log|z_2|^2}\left(|z_2|^4|z_3|^5 - |z_2|^5|z_3|^4\right) < 1\right\}$$

is a bounded domain with non-compact automorphism group and C^2-smooth boundary.

Similar examples can be constructed in any complex dimension for any $p \ge 3$ and $k \ge 1$. Note that there is considerable freedom in choosing a measure μ in (13). ∎

It is a reasonable question whether any function ψ as in Theorem 3 is given by formula (13) for an appropriate choice of μ. Note that, as shown in [FIK1], this holds if $k = \infty$, in which case the entries of $(p-1)$-tuples s from the set M can only be even integers and thus formula (13) turns into a polynomial (see (1)). However, as demonstrated by the following example, in the case of finite smoothness one can find functions that have the weighted homogeneity property, but that are not given by integration against a measure as in (13).

Example 6. As in Example 5, let again $n = 3$, $p = 3$ and $k = 2$. We set $\alpha_2 = \alpha_3 = 8$. Then it follows from (12) that

$$M = \{(2, 6)\} \cup \{(4, 4)\} \cup \{(6, 2)\}.$$

Since M is finite, the integral in (13) turns into a finite sum, and all functions of the form (13) are real-analytic. We are now going to present a C^2-smooth function $\psi(|z_2|, |z_3|)$ that has property (6) with $\alpha_2 = \alpha_3 = 8$ and such that ψ is not necessarily real-analytic.

Let $g \in C^2(\mathbb{R})$ be such that $g(0) = 0$ and $g(x) = x^2$ for $|x| > 1$. Then a direct calculation shows that

$$\psi(|z_2|, |z_3|) = |z_2|^8 + |z_3|^8 + |z_2|^8 g\left(\frac{|z_3|^2}{|z_2|^2}\right)$$

is C^2-smooth (for the last term $|z_2|^8 g\left(\frac{|z_3|^2}{|z_2|^2}\right)$ we set its value and the values of its first and second derivatives to be equal to zero whenever $z_2 = 0$). The above function ψ satisfies (6) with $\alpha_2 = \alpha_3 = 8$, but it, of course, is not real-analytic for any non-trivial choice of g. Also, if $g \geq 0$, one has that $\psi \geq 0$, and the corresponding domain (8) is bounded. The Reinhardt domain $D \subset \mathbb{C}^3$,

$$D = \left\{ |z_1|^2 + |z_2|^8 + |z_3|^8 + |z_2|^8 g\left(\frac{|z_3|^2}{|z_2|^2}\right) < 1 \right\} ,$$

is then also bounded and has a non-compact automorphism group and C^2-smooth boundary. Such an example can be given in any complex dimension for any $p \geq 3$, $k \geq 1$. ■

Example 6 shows that, most probably, a nice description of finitely smooth functions with weighted homogeneity property does not exist, at least in the form of an explicit formula such as (13). Therefore, Theorem 3 is likely to be the best possible classification result that one can hope to obtain for Reinhardt domains of finite smoothness.

It also may be noted that weighted homogeneous functions may be constructed by specifying them on the set $S_1 = \{|x_2|^{\alpha_2} + \cdots + |x_p|^{\alpha_p} = 1\}$ and then extending to all of the space by homogeneity as in the proof of Theorem 3 above (see (10)). Such a construction is useful in that it reduces the smoothness question to **(i)** checking smoothness on S_1; and **(ii)** checking smoothness at the origin (smoothness elsewhere is automatic).

Along the lines of the preceding discussion, one can consider the following examples of domains with non-compact automorphism group and C^k-smooth boundary, $k \geq 1$, that are not necessarily Reinhardt:

$$\left\{ |z_1|^2 + \psi(z_2, \ldots, z_n) < 1 \right\}, \tag{15}$$

where $\psi(z_2, \ldots, z_n)$ is a C^k-smooth function in $\mathbb{C}^{n-1}$ and

$$\psi\left(t^{\frac{1}{\alpha_2}} z_2, \ldots, t^{\frac{1}{\alpha_n}} z_n\right) = |t| \psi(z_2, \ldots, z_n) \tag{16}$$

in $\mathbb{C}^{n-1}$ for all $t \in \mathbb{C}$. Here $\alpha_j > 0$, $j = 2, \ldots, n$, and $t^{\frac{1}{\alpha_j}} = e^{\frac{1}{\alpha_j}(\log|t| + i \arg t)}$, for $t \neq 0$, where $-\pi < \arg t \leq \pi$. Also, to guarantee that domain (15) is bounded, one can assume that $\psi \geq 0$ and the domain

$$\left\{ (z_2, \ldots, z_n) \in \mathbb{C}^{n-1} : \psi(z_2, \ldots, z_n) < 1 \right\}$$

is bounded (cf. Theorem 3).

For any domain D of the form (15), $\mathrm{Aut}(D)$ is indeed non-compact, since it contains the subgroup

$$\begin{aligned} z_1 &\mapsto \frac{z_1 - a}{1 - \overline{a} z_1}, \\ z_j &\mapsto \frac{(1-|a|^2)^{\frac{1}{\alpha_j}} z_j}{(1-\overline{a} z_1)^{\frac{2}{\alpha_j}}}, \qquad j = 2, \dots, n, \end{aligned}$$

where $|a| < 1$. In addition to the above automorphisms, domains (15) are also invariant under the special rotations

$$\begin{aligned} z_1 &\mapsto e^{i\beta} z_1, \\ z_j &\mapsto e^{i\frac{\gamma}{\alpha_j}} z_j, \qquad j = 2, \dots, n, \end{aligned}$$

where $\beta \in \mathbb{R}$, $-\pi < \gamma \leq \pi$. Therefore, for any such domain D, one has $\dim \mathrm{Aut}(D) \geq 4$.

If $n = 2$, by differentiating both parts of (16) with respect to t and $\overline{t}$ and setting $t = 1$, one obtains that $\psi(z_2) = c|z_2|^\alpha$, with $c > 0$. Therefore, for $n = 2$, domain (15) is equivalent to a domain of the form (11) which is Reinhardt. However, as examples in [FIK2] show, there exist bounded domains in $\mathbb{C}^2$ with $C^{1,\beta}$-smooth boundary, for some $0 < \beta < 1$, with non-compact automorphism group, that are not biholomorphically equivalent to any Reinhardt domain and thus to any domain of the form (15). It would be interesting to know if, for $k \geq 2$, there also exist C^k-smooth bounded domains with non-compact automorphism group that are not equivalent to any domain (15), or, for $k \geq 2$, that the domains (15) are, in fact, the only possibilities up to biholomorphic equivalence.

For comparison, we state below the conjecture of Bedford/Pinchuk [BP2] (see also [BP1]) for domains with C^∞-smooth boundary. Assign weights $\alpha_j = 2m_j$, $m_j \in \mathbb{N}$, $j = 2, \dots, n$, to the variables $\tilde{z} = (z_2, \dots, z_n)$. If $K = (k_2, \dots, k_n)$ is a multi-index, we set $\mathrm{wt}(K) = \sum_{j=2}^n \frac{k_j}{\alpha_j}$. Consider real polynomials of the form

$$P(\tilde{z}, \overline{\tilde{z}}) = \sum_{\mathrm{wt}(K) = \mathrm{wt}(L) = \frac{1}{2}} a_{KL} z^K \overline{\tilde{z}}^L, \tag{17}$$

where $a_{KL} \in \mathbb{C}$ and $a_{KL} = \overline{a_{LK}}$.

Conjecture (Bedford/Pinchuk). Any bounded domain with non-compact automorphism group and C^∞-smooth boundary is biholomorphically equivalent to a domain

$$\left\{ |z_1|^2 + P(\tilde{z}, \overline{\tilde{z}}) < 1 \right\},$$

where P is a polynomial of the form (17).

The above conjecture was proved in [BP2] for convex domains of finite type and in [BP1] for pseudoconvex domains of finite type for which the Levi form of the boundary has rank at least $n-2$. Note that, for polynomials (17), as well as for functions ψ as in (5), condition (16) is satisfied.

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References

[**BP1**] Bedford, E., Pinchuk, S., *Domains in $\mathbb{C}^{n+1}$ with non-compact automorphism group*, J. Geom. Anal. **1** (1991), 165–191.

[**BP2**] Bedford, E., Pinchuk, S., *Convex domains with non-compact automorphism group (translated from Russian)*, Russian Acad. Sci. Sb. Math. **82** (1995), 1–20.

[**FIK1**] Fu, S., Isaev, A. V., Krantz, S. G., *Reinhardt domains with non-compact automorphism groups*, Math. Res. Letters **3** (1996), 109–122.

[**FIK2**] Fu, S., Isaev, A. V., Krantz, S. G., *Examples of domains with non-compact automorphism groups*, MSRI Preprint 1996-004, available at http://www.msri.org/MSRI-preprints/online/1996-004.html.

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